A review of Stock Pinning and its potential causes

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Business 801: Signature Project

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April, 2015

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Title of Signature Project: A review of Stock Pinning and its potential causes

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Degree: Master of Business Administration **Year:** 2015

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Abstract

This paper reviews the literature on stock pinning, discusses the relevant theoretical models and presents areas for future research. The literature on stock pinning makes the following conclusions. First, option trading impacts stock prices. Second, stock pinning is a real phenomenon, we see share prices cluster around particular strikes on option expiry days. Third, we see this phenomenon across a number of stocks. Fourth, non-optionable stocks are less likely to have their prices cluster around a predetermined price on any day. Finally, stock pinning is a product of delta-hedging by hedge traders.

A. Introduction

Researchers and practitioners have extensively studied stock market anomalies. Financial market participants investigate these anomalies with the hope of exploiting and profiting from them. Interestingly, many of these anomalies disappear over time. For instance, the weekend effect is no longer significant for various stock volatility indices (Cote, 2014). The existence of financial market anomalies, to the extent that they result in identifiable patterns is a paradox. If these anomalies are real, the Efficient Market Hypothesis (EMH) dictates that traders would quickly exploit them for a riskless profit, and prevent their reoccurrence. Still some anomalies continue to persist (e.g., stock pinning — the clustering of stock prices on certain days). This paper reviews the design, data, models, and findings of published research on this topic.

B. Introduction to Stock Pinning

The principle that prices, at least in the near term, are difficult, if not impossible to predict, is at the core of financial markets. Market participants determine prices by submitting buy and sell orders at random times. Thus, it is the interaction of many traders with diverse trading strategies that ultimately determines the price of a financial asset (Primbs, Rathinam, 2009). If so, mechanisms that influence and therefore make near term prices predictable should not exist. Yet, evidence of stock pinning is overwhelming.

Pinning refers to the likelihood that the price of a stock coincides with the *strike price*[1] of an *option* immediately before the expiration date of the latter (Avellaneda, Kasyan and Lipkin, 2012). Option trading is a zero-sum exercise (i.e., only the buyer or seller of a particular contract, not both, can benefit from the price trajectory of the underlying asset). The Chicago Board

[1] Appendix 8 provides a glossary of definitions for key words related to options.

Options Exchange (CBOE) introduced exchange-based option trading in 1973. The CBOE's pilot program initially limited trading to call options on only 16 underlying stocks. The CBOE conducted a price and volume analysis in 1976 and concluded that trading in these options had no significant effect on underlying stock prices. By June 1977, the CBOE had increased the number of optionable stocks to 219 and introduced put options for 25 of those stocks[2]

Since then, a number of price-impact studies have produced evidence that option trading alters the price of the underlying asset. Stock prices are not independent of the options that are written on them (Avellaneda, Kasyan and Lipkin, 2012). There is striking evidence that option trading changes the price of the underlying (Ni, Pearson and Poteshman, 2005). Specifically, empirical evidence has emerged of non-random stock price movement on option expiration dates. According to Ni, Pearson and Poteshman (2005), option writing explains the increased probability that stock prices cluster near particular strike prices. The authors also demonstrate that pinning results in average wealth transfers of over $9 billion on each expiration date.

Investigating stock pinning is important to stock market participants and regulators. The integrity of capital markets is at stake if the line between legitimate hedging practices and price manipulation are not clearly defined. Evidence that manipulation causes price clustering will trigger legislative response from regulatory bodies, and rightfully so.

C. Literature Search and Results

The University of Prince Edward's Island's Robertson library maintains the primary databases that I utilized to conduct an extensive literature search. The articles I review are from either the

[2] Securities and Exchange Commission. (1978). Report of the Special Study of the Options Markets to the Securities and Exchange Commission. US Government Printing Office, Washington, DC.

Business Source Complete or Academic Source Complete databases. A search for additional literature on Google Scholar did not yield any articles not already on the University's databases (see Appendix 1). In all, the search yielded one robust empirical study on stock pinning (Ni, Pearson and Poteshman, 2005). Stoll and Whaley (1986, 1987, 1991, 1997), Golez and Jackwerth (2010) explore pinning but only in relation to index options and futures. Perhaps a lack of interest due to the relatively small size of the options market in comparison to equity, fixed income and commodity markets explains the scarcity of original empirical studies. The high cost of acquiring historical option trading data is another plausible explanation, albeit a less likely one because rational investors will readily incur cost if the expected return exceeds the expense. However, the results of Ni, Pearson and Poteshman (2005) form the basis for a number of mathematical stock pinning models [e.g., Nayak (2007), Jeannin, Iori and Samuel (2008) and Avellaneda, Kaysan and Lipkin (2012)].

D. An Overview of Important Concepts

i. Option Trading Strategies

Long strategies result in ownership of the derivative contract. Ownership of an option grants the holder the right buy or sell the underlying asset at a fixed price (strike price). The owner benefits if the market price of the underlying differs from the fixed price, such that a subsequent transaction to buy or sell of the underlying generates a profit. The owner pays for this right by depositing a premium. Owners of option contracts lose their premium if the subsequent transaction will result in a loss. Recall that option trading is a zero-sum game; therefore losses to the owners of option contracts accrue as gains to the sellers of these contracts. A seller of an option contract is *short* the contract and profits by keeping some or the entire premium paid by the buyer (see Appendix 4 for option value payoffs)

ii. The Greeks: Delta, Gamma and Theta

The *delta* of an option is the change in value of the option for a change in value of the underlying asset. For example, assume stock X is trading at $100 and an *at the money* call option for stock X has a delta of 0.4. If the price of stock X increases by $1 to $101, then the value of the *call option* will increase by $0.40 per share. One option contract represents interest in 100 shares, therefore the market value of the option contract will increase by $40 (100×$0.40). The delta of an option changes continuously based on market conditions. The rate of change of delta is measured by *Gamma*. Net buyers of options face a positive gamma, that is, delta increases by gamma for each dollar increase in the value of the underlying. The larger the dollar increase in the underlying the higher the rate of change of delta (i.e. the higher the gamma). More pointedly, for long positions in calls, *puts* or *straddles*, gamma is positively correlated to price increases of the underlying. The opposite is true for short option positions (See Figure 1).

Strategy	**Position Gamma Signs**
Long Call	Positive
Short Call	Negative
Long Put	Positive
Short Put	Negative
Long Straddle	Positive
Short Straddle	Negative

Figure 1 - Position Gamma signs for common strategies for options[3]

The value of an option (American option) consists of intrinsic value and time value. Intrinsic value is the difference between the strike and current price of the underlying. (i.e., [S – X] for

[3] See http://www.investopedia.com/university/option-greeks/greeks5.asp

calls and[X – S] for puts, where S is current price and X is the strike price)[4]. The intrinsic value of an option changes as the price of the underlying changes. The time value is a dollar measure of the probability that the value of the option increases before the option expires. As time passes and the option approaches maturity, the time value of an option decreases. This decrease in time value is referred to as time decay and is measured by the Greek letter *theta*. As I will later discuss, high theta favours sellers of options, specifically those engaging in stock pinning activities.

iii. Delta Hedging

Consider a trader who purchases one call option (long call) on stock X mentioned above. A $1 move increases his portfolio by $40 (his portfolio has a delta of 40). The trader will hedge his portfolio by short-selling 40 shares of stock X. If stock X falls by $1, the $40 loss in the trader's option will be offset by a $40 gain in the short equity position. This practice is referred to as *delta hedging*. The approximate change in option price for a given change in price of the underlying is less accurate for large movements in price. That is, as the rate of change in delta increases (gamma increases) it becomes difficult to approximate the number of shares of the underlying required to perfectly hedge a portfolio and delta hedging becomes less effective.

iv. Geometric Brownian Motion

Brownian movement (motion), first noted by botanist Robert Brown is used across various disciplines to describe any physical phenomena in which a given quantity constantly undergoes small, random fluctuations[5]. Geometric Brownian motion is a continuous random process in

[4] Appendix 4 summarizes option payoffs (at expiration) for American and European options.

[5] See http://www.columbia.edu/~ks20/FE-Notes/4700-07-Notes-BM.pdf

which the logarithm of a random variable follows a Brownian motion. In our case, the random variable is the future stock price and its logarithm is price change. The underlying theory holds that price changes are random (Brownian movement) and follow a normal distribution (see Figure 2). The implication is that future stock prices are lognormally distributed - a distribution with a positive skew to the right and bounded by zero to the left (see Figure 3). The theory is consistent with the observation that stock prices cannot fall below zero.

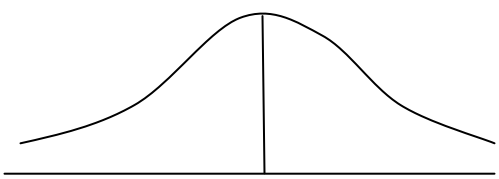

Figure 2: Normal distribution (Price changes)

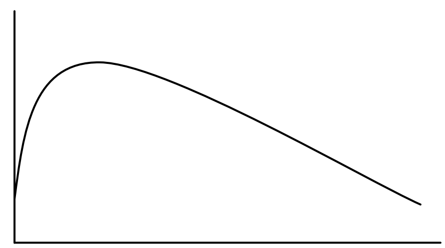

Figure 3: Lognormal distribution (Future stock price)

E. Stock Pinning: Data & Design Issues

Ni, Pearson and Poteshman (2005) make a compelling case for the need for an extensive, empirical study investigating stock pinning. Prior to their article only a handful of studies existed, all of which were without depth. For instance, Klemkosky (1978) find statistical difference between stock returns in the week leading up to expiration and the week after expiration. However, Klemkosky’s study covered only 14 expiration dates, an insufficient length of time to establish a trend. Cinar and Vu (1987) remedy this by conducting a longer study, analyzing data over a six and half year period. Cinar and Vu find inconclusive empirical evidence of stock pinning but their sample size of only six stocks brings into question the validity of their study. Krishnan and Nelken (2001) provide evidence that shares of Microsoft clustered near integer multiples of $5 more regularly on expiration that on any other days. In contrast Ni, Pearson and Poteshman (2005) analyzed 16,396 optionable and non-optionable stocks across 80 expiration dates over a six and half year timespan (January 1996 to August 2002). There were approximately 2,500 optionable stocks at any given point in the duration of the study period.

The study design is robust, of the 16,396 stocks analyzed 4,396 are optionable stocks and 12,001 are non-optionable stocks. If pinning is real, price clustering should be evident in the optionable stocks and absent in non-optionable stocks. Further, price clustering on option expiration days should disappear when an optionable stock becomes non-optionable and appear when non-optionable stock becomes optionable. Ni, Pearson and Poteshman (2005) cross-examine the 16,396 against these criteria and find evidence of stock pinning. They make a significant contribution because stock pinning has real economic implications. Wealth transfers from option buyers to option sellers when options expire worthless as a consequence of pinning. Moreover, all holders of the underlying stock, including those who do not trade the derivatives, see their returns altered on option expiration days.

After discovery, the primary concern becomes the root cause of the pinning. The prevailing proposition across the literature is that stock pinning is a product delta-hedging trading. Avellaneda and Lipkin (2003) conclude that pinning can be caused by floor traders who are delta-hedging long-gamma positions on a particular strike with higher than normal *open interest*[6]. Jeannin, Iori and Samuel (2008) analyze the feedback effects of option hedging strategies and show that that *dynamic hedging* is responsible for the pinning effect. Nayak (2007)

[6] Long-gamma position implies that a trader is a net buyer of calls, puts or a straddle. If the particular long position has higher than usual open interest, the trader's transactions to arrive at a delta-neutral portfolio can cause stock prices to cluster around that position's strike price.

finds that hedgers who are long a *straddle* position tend the push the underlying stock toward the strike. In addition to the delta-hedging hypotheses, Ni, Pearson and Poteshman (2005) investigate two more possible explanations. The first is that, price clustering is the product of investors closing out certain combinations of stock and option positions. The second is that firm proprietary traders, who have written a large number or options, manipulate the price of the underlying stock so that it closes at the target strike[7] or just outside the target strike, preventing the options from being exercised. These traders profit by keeping the premiums paid by the buyer for the contract.

Other studies offer alternative explanations for stock price clustering. Brown, Chua and Mitchell (2002) study cultural biases on asset prices. Bita (1997) provide evidence that “feng shui’ and other Chinese superstitions impacts real estate prices. Brown, Chua and Mitchell (2002) extend this research to six Asia-Pacific equity markets (Australia, Hong Kong, Indonesia, Philippines, Singapore and Taiwan). They find that stock prices are likely to cluster around whole dollar amounts ending in zero, five, and even integers. Price clustering around these numbers is found to be especially significant in the Hong Kong market around Chinese New Year, the Dragon Boat and the Mid-Autumn festivals.

Grossman, Miller, Cone, Fischel and Ross (1997) study London Stock Exchange quotes over a one month period (October 1994) and find an increased likelihood that price clustered around zero and five than around any other integers. A majority of US listed options studied by Ni, Pearson and Poteshman (2005) also have strike prices rounded to zero or five. Thus we can

[7] In most cases it is the strike price closest to Thursday’s closing price.

speculate that US listed stock prices also cluster around zero and five. This is relevant because the distance between consecutive option strike prices impacts hedging strategy[8].

Regarding price clustering, Grossman, Miller, Cone, Fischel and Ross (1997) and Brown, Chua and Mitchell (2002) find a general attraction to whole numbers, but neither one investigates a difference in return on a particular date versus all other dates. Brown, Chua and Mitchell (2002) find increased clustering around certain holidays but attribute it solely to the cultural bias of market participants. Ni, Pearson and Poteshman's study differs in that it explores market activity as a cause of price clustering; Grossman, Miller, Cone, Fischel and Ross (1997) and Brown, Chua and Mitchell (2002) investigate the phenomena as a product of subconscious human tendencies.

A thorough review of Ni, Pearson and Poteshman's landmark study is imperative because it is the most comprehensive empirical study of stock pinning[9]. Consequently it is referenced across multiple studies as primary evidence of stock price clustering on/near option expiration dates.

> "…conclusive evidence of stock pinning near option expiration dates was given by Ni, Pearson and Poteshman (2005)." – Avellaneda, Kaysan and Lipkin (2012, pp. 949).
>
> "…Ni, Pearson and Poteshman (2005) present further compelling evidence of stock price clustering at expiration dates." – Nayak (2007 pp. 536)
>
> "…A thorough analysis of the pinning effect has been provided by Ni, Pearson and Poteshman (2005)" – Jeannin, Iori and Samuel (2008 pp.823)

F. Empirical Evidence of Stock Pinning

Ni, Pearson and Poteshman (2005) is a correlation study designed to determine the relationship between option trading and stock prices on option expiry dates. The authors gather

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[8] Implied volatility and gamma risk is reduced when the difference in consecutive strike prices is smaller.

[9] To the best of my knowledge

extensive data on all stocks and monthly options traded in US exchanges between January 1996 and September 2002. Specifically, they acquired from Ivy DB (option metrics) database, end of day bid-ask quotes and volumes for all exchange traded stocks and bid-ask quotes, volumes and open interest for all exchange traded options. Additionally they collected open interest and trading volume from CBOE for the period January 1996 to December 2001. The CBOE statistics were further broken down into investor category types, namely Market Makers, Firm Prop Traders, Large Firm Clients and Discount Firm Clients. The study is limited to exchange transactions and makes no mention of trades conducted in *dark pools*, where a sizable quantity of institutional trading takes place. As of 2012 approximately one third of all equity trades occurred in a dark pool[10].

Further, only the CBOE statistics categorize investor types. CBOE accounts for only 28% of US option trading volume[11], meaning that investor category breakdown is unavailable for majority of the data. Thus I view with scrutiny any conclusions made in the study regarding the price-impact of a particular type of investor. For instance, the claim of evidence of manipulation by firm proprietary traders may be inaccurate since the origin of trades was only known for less than a third of the data. Although this design flaw does not impact the conclusion that pinning is a real phenomenon, it may overstate manipulation as a covariant and understate other variables that may be related to pinning.

Beyond that, Ni, Pearson and Poteshman studied thousands of optionable and non-optionable stocks over at least eighty expiration dates (monthly US exchanged traded stock options expire

[10] See http://www.bloomberg.com/bw/articles/2012-05-10/where-has-all-the-stock-trading-gone

[11] See http://ir.cboe.com/press-releases/2014/jun-2-2014a.aspx

on the third Friday of the month). The study found that approximately 19% of optionable stocks close within \$0.25 of a strike price, 11.5% close within \$0.125 of the strike and 3.2% close at a strike price on expiration date. In all cases the percentages are significantly higher on expiration Friday than on any other date. Closing prices of non-optionable stocks were not found to be statistically different on expiration Friday than on any other day. For brevity, the authors choose to focus on cases of stock prices closing within \$0.125 of the strike.

The study omits (but it is worth noting) that the Options Clearing Corporation (OCC) will generally automatically exercise options that are more than \$0.01 in the money[12]. For most investors, transaction costs may exceed the gain from exercising a \$0.01 in the money option. Consequently, this paper considers a stock to be 'pinned' if its closing price at options expiry coincides exactly with a particular strike or if its price is within a range where potential gains from exercising an in the money option are outweighed by transaction costs. Therefore I agree with the decision to focus on closing prices within +/- \$0.125 of a target strike, albeit for different reasons.

G. Stock Pinning: Manipulation by Market Makers?

Kraft and Kuhn (2011) study the price impact of large traders on illiquid derivatives. They conclude that large traders can manipulate a derivative's payoffs in the direction that is favorable for them. Carhart, Kaniel, Musto, and Reed (2002) find that towards the end of calendar quarters and years, mutual fund managers manipulate the prices of their equity holdings. They do this because industry rankings and benchmark comparisons are based on data at the end of these periods. These studies demonstrate the existence of market participants who have intent and

[12] See http://www.cboe.com/learncenter/concepts/beyond/expiration.aspx

ability to manipulate asset prices. Ni, Pearson and Poteshman (2005) explore the hypothesis that some option traders may be such participants.

Ni, Pearson and Poteshman (2005) provide no empirical evidence for or against price manipulation by market makers as the cause of stock pinning. Instead, they provide a qualitative assessment of why it cannot be the primary cause. Manipulation is costly, therefore the incentive to constrain price should disappear when the underlying stock reaches a price such that options around the target strike price are unlikely to be exercised. The underlying assumption is that option market makers, who have net sold positions, are most likely to manipulate price and will continue to do so until written options are just out the money (OTM). If true, stock prices would likely settle at prices close to the strike (because market makers cease manipulation at this price). Logically, it follows that market makers with net purchased option positions have incentive to manipulate prices such that their option positions close in the money (ITM). However Ni, Pearson and Poteshman argue that this latter scenario is unlikely. They explain as follows: if a call option buyer manipulates prices upwards so that he exercises his call option, then he will receive share of an overpriced stock which may be difficult to unload. Similarly if a put option buyer manipulates prices downwards so that she exercises her put options, then she will sell shares of an underpriced stock, diminishing any potential profits. Further, traders with net purchased positions have added incentive to continue manipulating stock prices after the options become ITM because they profit the deeper ITM the option is. Such manipulation, if it exists, would push stock prices further away from the strike rather than produce clustering around it. A more practical reason that dissuades option market makers from manipulating stock prices is that their transactions in the underlying stocks are closely monitored by the exchange (Cox and Rubinstein, 1985)

H. Stock Pinning: Manipulation by Firm Proprietary Traders and Public?

Assembling trading data to investigate market phenomena can be routine, but the intent behind the trades at the time of execution is often difficult to discern. Ni, Pearson and Poteshman (2005) address this by analyzing trading volume towards option expiry. They argue that market participants wishing to manipulate stock prices will likely initiate option positions days before the expiration date. Investors with large written open interest positions also have an incentive to begin manipulating prices as the expiration date approaches regardless of the time or reason that they initiated the trade. If this is the case, we should expect to see a spike in open interest of options closest to the current price as investors position themselves to profit from constraining the stock price (Ni, Pearson and Poteshman, 2005). A short-coming of this argument is that market participants employ a variety of fundamental and technical trading strategies, some of which may entail increasing purchases of various derivatives as the standard equity options expiration date approaches (every 3rd Friday of the month). It may be incorrect to assume that new buying and writing of options on expiration week signals intent to manipulate.

Even so, the assumption is not entirely without merit. Given the monetary costs of continuous trading to manipulate prices, participants willing to engage in such practices would seek to minimize these costs by entering the minimum number of trades required to achieve their objectives. One way to do this would be to wait until the option expiration date was near before initiating new positions. Further, the risk of detection is lower the shorter the period of time over which the manipulation occurs.

Ni, Pearson and Poteshman (2005) put forth that successful manipulators would need to have considerable resources and access to privileged information. Members of the public lack the resources and insider information required to manipulate stock prices. Firm proprietary traders, on the other hand, have the means, inside information, and face only a fraction of the scrutiny that market makers are subjected to. They have an incentive to write a large number of close to the money or ATM options and then manipulate the underlying price such that these options close OTM.

Consider the case of Apple Inc. on March 20th 2015. As expiration approaches, open interest is higher on the $125 strike for weekly expiration calls and puts, than on any other strike (see Appendix 5). The claim is that manipulators are net short a large number of calls and puts at the $125 strike and stand to benefit if the stock is pinned at that price. Approximately, ten minutes before the close, Apple is trading at $127.75. Without warning, Apple's stock price falls to a low of $125.16, a two percent decline in nine minutes[13] (see Appendix 6). By comparison, Apple's peers, the next five largest companies[14] (by market capitalization) in the NASDAQ 100 index declined an average of only -0.143% in the same time period (see Appendix 7). The S&P 500 broad market index fell -0.168%, noteworthy because Apple has a *beta* coefficient of 0.94 (i.e., Apple's price movements are highly correlated with the S&P 500 index). Seconds after closing at $125.90, Apple's after-market bid jumps to $126.85, a 0.76% increase, implying that Apple's price may have been artificially suppressed towards market close (see Appendix 8).

[13] There was no news regarding Apple Inc. or any of its products.

[14] Microsoft Corp, Google Inc., Facebook Inc., Amazon Inc., Intel Corp. respectively.

Unlike delta hedging strategies that apply price pressure from both above and below the strike, firm proprietary traders would have a skewed impact on price. Firms that are net writers of puts would manipulate to keep prices above the target strike and firms that are net writers of calls would manipulate to keep prices below a target strike.

I. Stock Pinning: Modelling Issues – The Hedge Rebalancing Effect

To explain the consensus cause of stock pinning, I begin by visiting the composition of market participants. Nayak (2007) classifies investors into two groups, reference traders and hedgers. The former, trades on a belief in the true value of a stock, while the latter are option holders who buy and sell stock consistent with the Black-Scholes-Merton model. Primbs and Rathinam (2009) suggest a more comprehensive classification of traders that includes extraneous, value, momentum and hedge traders. The first three are reference traders under Nayak's (2007) classification system. Extraneous, value and momentum traders utilize information that they perceive to be insightful to make buy/sell decisions. Hedge traders seek to eliminate portfolio risk.

There are several mathematical models that describe stock pinning. Avellaneda and Lipkin (2003) propose a price-impact model where a linear price-elasticity equation drives the price of the underlying stock (see Equation 1). That is, changes in the underlying stock price follow changes in supply/demand of the stock, which is driven by hedgers who are hedging option positions that are sensitive to price movements in the underlying. These price changes follow a normal distribution per the Black-Scholes-Merton equation (see Equation 2).

Equation 1: Avellaneda-Lipkin linear price-impact model[15]

$\Delta S/S = EQ$

ΔS = change in stock price S = Stock price. E = Elasticity of demand (constant) Q = quantity of stock demanded.

Equation 2: Black-Scholes-Merton Formula (for European Call and Put options)[16]

$$c = S_0 N(d_1) - Xe^{-r^c T} N(d_2)$$

$$p = Xe^{-r^c T}\left[1 - N(d_2)\right] - S_0\left[1 - N(d_1)\right]$$

$$d_1 = \frac{\ln(S_0/X) + \left[r^c + (\sigma^2/2)\right]T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

c = Call premium

p = Put premium

S_0 = Current Stock Price

T = Time until option exercise

X = Option strike price

r^c = Risk-free interest rate (continuously compounded

N = Cumulative Standard normal distribution

e = Exponential term

σ = Standard deviation

ln = Natural log

The use of Black-Scholes assumptions by Avellaneda and Lipkin (2003) handicaps the model, as far as real markets application is concerned. These four assumptions form the basis of the Black-Scholes model (Jarrow, Wiggins, 1989), for a European option:

i. Frictionless markets

ii. Constant risk free rate and volatility

iii. No dividends on underlying over life of the option

iv. Price changes are normally distributed and follow a random walk.

[15] Avellaneda, M., & Lipkin, M. D. (2003). A market-induced mechanism for stock pinning. *Quantitative Finance* (p. 419)

[16] Institute, CFA. 2015. CFA Level II Volume 6 Derivatives and Portfolio Management. Wiley Global Finance, 2014-07-14. VitalBook File.

The normally distributed return assumption implies a lognormal distribution of the underlying asset price which is contradictory to the findings of empirical work (Sabanis, 2003). Secondly, volatility is stochastic (Hull and White, 1987) not constant as per the Black-Scholes equation. Further, Avellaneda and Lipkin (2003) concludes that stock pinning is a real phenomenon. If so, it follows that underlying asset prices are different from a *random walk*, therefore any models (including Avellaneda and Lipkin's) built on non-deterministic principles are flawed. Other authors have attempted to remedy some of these short-comings. Stock price changes do not follow the bell curve (Mandelbrot, 2004) and so lognormal price models have limitations in live markets.

Nayak (2007) follows the work of Ni, Pearson and Poteshman that pinning is real and is caused by hedgers. Nayak releases the log-normal price assumption and proposes an equilibrium-based model in which the interaction of reference traders and hedgers determines price (recall Avellaneda and Lipkin's model where price of underlying is driven primarily by the actions of hedgers). Reference traders demand and supply shares based on perceived differences between real value and market value of stocks. Hedgers enter into a straddle position (equal quantity of puts and calls at the same strike) and then buy or sell to maintain a riskless portfolio. When hedge traders are net long options (i.e., when they are net owners of options), they become long gamma. Stated simply, they lose money on a fast movement in the price of the underlying. Therefore they hedge by buying when price is decreasing (or below the strike) and selling when price is increasing (above the strike), causing pressure on the stock from above and below (Avellaneda and Lipkin 2003).

Despite the differences in approach, both models (Avellaneda and Lipkin's and Nayak's) arrive at a similar conclusion. To demonstrate, note the following two equations.

Equation 3: Avellaneda and Lipkin (2003). Price Impact Model: Stochastic differential equation[17].

$$dy = -\frac{nE}{\sqrt{2\pi}}\frac{y - a(T-t)}{\sigma(T-t)^{3/2}} e^{-\frac{(y+a(T-t))^2}{2\sigma^2(T-t)}}\, dt + \sigma\, dW.$$

n = number of straddle positions entered into by hedgers. W = Standard Brownian motion

Equation 4: Nayak (2007). Equilibrium Based Model[18]

$$-\beta\left(\frac{dS_t}{S_t} - \frac{dX_t}{X_t}\right) - nd\Delta = 0.$$

n = number of straddle positions entered into by hedgers.

Detailed analysis of equations 3 and 4 is outside the scope of this review. I draw attention to the variable denoted *n* in both models. For Avellaneda and Lipkin (2003), when *n=0*, the equation reduces to $dy = \sigma dW$ which has a solution that follows a geometric Brownian motion (Avellaneda and Lipkin, 2003). Similarly, when *n=0*, the underlying stock price dynamics are the usual geometric Brownian Motion (Nayak, 2007). The key insight is that when there is no hedging activity in the marketplace (n=0), changes in stock price follow a random walk (Brownian Motion) and pinning cannot be a real phenomenon (i.e., pinning is evidence of non-random prices).

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[17] Avellaneda, M., & Lipkin, M. D. (2003). A market-induced mechanism for stock pinning. *Quantitative Finance* (p. 419)

[18] Nayak, S. (2007). An equilibrium-based model of stock-pinning. *International Journal of Theoretical and Applied Finance* (P. 539)

When $n \neq 0$, hedgers are present, they are either net long or net short options, stock prices differ from a random walk and pinning can be a real phenomenon. For further clarity, I explore the work of Jeannin, Iori and Samuel (2008). Their model extends Avellaneda and Lipkin's (2003) and defines a variable $d\Delta$ as the amount of stock required to be bought and sold to hedge a net long options position as time goes by. Their findings are consistent with Avellaneda and Lipkin (2003) and rely on Ni, Pearson and Poteshman (2005) for empirical evidence of stock pinning. When stock price is above the straddled strike, $d\Delta$ is negative which forces the hedger to sell, thereby pushing the price downwards towards the strike. The opposite is also true. When $d\Delta$ is positive, the hedger is induced to buy, pushing the stock price up towards the strike (Jeannin, Iori and Samuel, 2008)

I did not extensively review other models for this paper [e.g., Frey and Stremme (1997), Sircar and Papanicolaou (1998), Schonbucher and Wilmott (2000), Platen and Schweizer (1998)],but all conclude that stock returns are not normally distributed when hedgers are present.

J. Stock Pinning Modelling Issues – The Manipulation Effect

Recall the proposition by Ni, Pearson and Poteshman (2005) that firm traders are solely responsible for the manipulation effect and that their impact on price is skewed by their net call or net put position. Firm proprietary traders are interested only in the profitability of their options portfolio and will intervene to manipulate by either buying or selling the underlying, skewing price to one direction. Contrast this to market-makers who are hedging a portfolio consisting of both the option and the underlying. They intervene to hedge by buying and selling to arrive at a risk free portfolio. Ni, Pearson and Poteshman (2005) model manipulation following the conditional probability below:

Probability that Friday close is pinned just below Thursday's close, given that Thursday's close was above the target strike, denoted by $P\{S_{Fri}\ E[K – 0.125, K] \mid S_{Thurs} > K\}$ less the probability that Friday close is pinned just above Thursday's close, given that Thursday's close was below the strike, denoted by $P\{S_{Fri}\ E[K, K + 0.125] \mid S_{Thurs} < K\}$.

Equation 5: Ni, Pearson and Poteshman (2005): The Manipulation Mechanism[19]

$$P\{S_{Fri}\ E[K – 0.125, K] \mid S_{Thurs} > K\} - P\{S_{Fri}\ E[K, K + 0.125] \mid S_{Thurs} < K\}$$

P = Probability of event

S_{Thurs} = Thursday's closing price

S_{Fri} = Friday's closing price

K = Target Strike

*Note, stock price is considered 'pinned' if it closes at target strike or +/- 0.125 of target strike

Ni, Pearson and Poteshman (2005) argue that absent manipulation, the effect of delta hedging will impact both sides of the expression equally, because hedgers both buy and sell to maintain delta neutrality. On the other hand, manipulation will cause the quantity (equation 5) to increase either by increasing the left side of the expression or by decreasing the right side of the expression. The left side increases when manipulators who are net call writers, have an increased probability of selling the underlying to drive down price when Thursday's price is above the strike. The right side decreases when net call writers decrease the probability that Friday's price goes up past the strike if it closed below it on Thursday.

The primary issue with Ni, Pearson and Poteshman's approach is that it generalizes firm proprietary traders as one market participant trading to maximize profits for one portfolio. In reality, firm A may be net call writers and interested in manipulating the price downwards past the strike. Firm B, may be net put writers and may be interested in manipulating the price

[19] Xiaoyan Ni, S., Pearson, N. D., & Poteshman, A. M. (2005). Stock price clustering on option expiration dates. *Journal of Financial Economics (P.84)*

upwards past the strike. Assuming both firms have equal or near equal skills and resources, as is the case in reality, neither one will be successful in achieving their goals and would therefore cease further attempts. Secondly, to successfully manipulate prices in one direction, the two firms would have to pre-emptively collude to be net writers of calls or puts, an unlikely prospect given the risk of detection and magnitude of the sure repercussions. Lastly, the vast majority of proprietary trading firms are small boutique shops without the resources to significantly impact stock prices.

The authors find price manipulation to be significant at the 5% level of significance. I contend that manipulation may be statistically significant only because of the model design. A more thorough study would identify and track trades placed by the large firm proprietary traders in the week leading to expiry. Further, we would require access to trades taking place inside dark pools, in order to determine definitively whether a large firm was buying or selling a particular stock to manipulate its price in favour of the firm's own option portfolio. Identifying and tracking option trades by large firm traders is conceivable since CBOE and other option exchange data is available publicly or for purchase. However, acquiring data on trades conducted inside dark pools may be virtually impossible. Dark pools are designed specifically to mask the identity of institutions trading within them.

K. Conclusions and Suggestions for Further Research

The literature on stock pinning affirms that stock pinning is a real phenomenon caused primarily by hedgers trading to maintain delta neutral portfolios. Pinning is most likely to occur when there is an unusually high open interest in the strike closest to Thursday's closing price and when delta-hedging traders have long-gamma positions on that particular strike. Delta hedging influences price from both above and below the target strike, causing the stock price to be pinned at that particular price. Research also finds that stock price manipulation by firm proprietary traders contributes to price clustering on option expiry dates. Lastly, in most cases pinning occurs when stock prices remain in the neighbourhood of their Thursday's closing prices, as opposed to being driven up or down towards the target strike when prices are distant from Thursday's close.

This review raises specific reservations about the available research. First, the mathematical formulae used to model stock pinning rely on theoretical assumptions that are false in real markets. Volatility changes over time and these changes are random, not constant as per Black-Scholes. Stock returns are not necessarily normally distributed, so it follows that future price changes may not be lognormal, thus using the Geometric Brownian Motion to model future price may be incorrect. Lastly, regarding the claim of manipulation by firm proprietary traders, it is difficult to quantify intent to manipulate based purely on trading volume data.

Future research should investigate the impact of weekly options on stock pinning. The introduction of weekly options may have created a decline in the open interest of monthly expiry options. If so, it is worthwhile to investigate whether pinning persists in the absence of unusually high open interest in the monthlies, or whether pinning now occurs on a weekly basis. Future research should focus on improving the predictive value of current stock pinning models. Delta-hedging may indeed explain some of the variance in stock returns on option expiry dates but it is difficult to say with certainty that it is the primary cause of pinning while relying on post hoc mathematical models.

Research should also test the hypothesis that the probability of pinning is higher in stocks with a relatively small difference[20] in consecutive option strike prices, than in those with a larger difference[21] in consecutive strike prices. A smaller difference is indicative of smaller absolute moves in the price of the underlying and consequently a lower gamma (gamma is positively correlated to price changes in the underlying). Investors seeking to take advantage of this relatively lower volatility are likely to initiate short positions. If market makers have to take the opposite side of the trade, they will be net long options in a low gamma portfolio. Low gamma risk implies a stable delta and increases the effectiveness of delta hedging. Research could explore whether this increased effectiveness translates into a higher likelihood of pinning.

[20] Las Vegas Sands (LVS) has $2.50 between consecutive strike prices for June 19 2015 expiration. http://finance.yahoo.com/q/op?s=LVS&date=1434672000

[21] Apple Inc (AAPL) has $5.00 between consecutive strike prices for June 19 2015 expiration. http://finance.yahoo.com/q/op?s=AAPL&date=1434672000

Appendix 1: Search Terms and Results

Database	Search term	AND	Constraints	Results	Relevant
Business Source Complete	Stock Pinning		N/A	31	10
	Stock Pinning	Option Expiration	N/A	4	4
	Price Clustering		N/A	527	N/A
	Price Clustering	Option Expiration	N/A	3	3
	Poteshman (AU)		Academic Journals	12	12
	Poteshman (AU)	Pearson (AU)	N/A	3	3
	Avellaneda (AU)	Pinning	N/A	1	1
	Avellaneda (AU)	Lipkin (AU)	N/A	2	2

Database	Search term	AND	Constraints	Results	Relevant
Academic Source Complete	Stock Pinning		N/A	17	5
	Stock Pinning	Option Expiration	N/A	1	1
	Price Clustering		N/A	352	N/A
	Price Clustering	Option Expiration	N/A	0	0
	Poteshman (AU)		N/A	0	0
	Poteshman (AU)	Pearson (AU)	N/A	0	0
	Avellaneda (AU)	Pinning	N/A	0	0
	Avellaneda (AU)	Lipkin (AU)	N/A	0	0

Database	Search term	AND	Constraints	Results	Relevant
Google Scholar	Stock Pinning	Price Clustering	N/A	15100	N/A
	Stock Pinning	Option Expiration	N/A	19500	N/A

**AU = Author*

Appendix 2: Reviewed Articles

Authors	Title	Summary of Findings	Limitations of Research	Contribution to Field
Avellaneda and Lipkin (2003)	A market-induced mechanism for stock pinning	The authors propose a model to explain stock pinning. They find that pinning can be the product of delta hedging by traders with large portfolio of written calls and puts. This study assumes a linear price impact function.	Model leans heavily on Black Scholes formula which has some incorrect real market assumptions.	Multiple other studies have built stock pinning models by extending Avellaneda and Lipkin's original model.
Ni, Pearson and Poteshman (2005)	Stock price clustering on option expiration dates	Empirical study. Found evidence of stock pinning. Attributes the phenomena to delta-hedging by market makers and manipulation by firm proprietary traders.	The data, specifically as it pertains to the claim of manipulation by firm proprietary traders is incomplete. The price impact of trades conducted in a dark pools is ignored.	This is the only robust empirical study of stock pinning over the last 10 years. (a thorough search across multiple databases yielded no other empirical studies.)
Nayak (2007)	An equilibrium-based model of stock-pinning	Prices are less likely to follow a simple geometric Brownian Motion if traders have predominantly net long option portfolios.	Ironically Nayak's model may be impractical in real markets even though its assumptions are closest to real markets. (for instance, future prices are not lognormal). Reason being that equity options are priced using Black Scholes assumptions, however incorrect they may be. The effectiveness of Nayak's model is diminished because it is not in wide use.	Introduce a significant shift from Avellaneda and Lipkin's (2003) model which summarizes that pinning is a product of the actions of hedge traders. Nayak puts forth that pinning is the result of the interaction of hedgers and reference traders (those seeking to profit from asset mispricing)

Jeannin, Iori and Samuel (2008)	Modeling stock pinning	Authors extend Avellaneda and Lipkin (2003) and show that delta hedging can push stock price towards the target strike and similar hedging can keep the stock price in the vicinity of the target strike.	Authors are extending Avellaneda and Lipkin (2003) model which has aforementioned real market limitations. They also introduce a revised Frey and Stremme (1998) model which assumes that hedgers do not consider the price impact of their own hedging transactions. In reality, they do.	Study compares various models and affirms that price approaches and stays at the target strike primarily due to delta-hedging.
Avellaneda, Kaysan and Lipkin (2012)	Mathematical models for stock pinning near option expiration dates.	The authors extend their original model, this time considering non-linear price impact functions.	Aside from the price functions, the authors fail to release any other core assumptions from their original model. They still rely heavily on the Black Scholes equation.	Study shows that pinning can be modelled even with non-linear price impact functions.

Appendix 3: CBOE Market Share[22]

CBOE Market Share Current Month (May 2014)						Year-To-Date		
	May-14	May-13	% Pt. Chg	April-14	% Pt. Chg	May-14	May-13	%Pt.Chg
Total Exchange	28.0%	24.6%	3.4%	27.2%	0.8%	28.3%	24.1%	4.2%
Equity Options	20.4%	16.5%	3.9%	20.1%	0.3%	21.0%	16.2%	4.8%
Index Options	96.8%	97.2%	-0.4%	96.0%	0.8%	96.8%	95.3%	1.5%
ETP Options	20.3%	18.9%	1.4%	21.0%	-0.7%	20.5%	17.4%	3.1%

Appendix 4: Option Values at Expiration (Payoffs)[23]

		Example (X = 50)	
Option	**Value**	**S_T = 52**	**S_T = 48**
Long European call	$c_T = \text{Max}(0, S_T - X)$	$c_T = \text{Max}(0, 52 - 50) = 2$	$c_T = \text{Max}(0, 48 - 50) = 0$
Long American call	$C_T = \text{Max}(0, S_T - X)$	$C_T = \text{Max}(0, 52 - 50) = 2$	$C_T = \text{Max}(0, 48 - 50) = 0$
Long European put	$p_T = \text{Max}(0, X - S_T)$	$p_T = \text{Max}(0, 50 - 52) = 0$	$p_T = \text{Max}(0, 50 - 48) = 2$
Long American put	$P_T = \text{Max}(0, X - S_T)$	$P_T = \text{Max}(0, 50 - 52) = 0$	$P_T = \text{Max}(0, 50 - 48) = 2$
Short European call	$c_T = -(S_T - X)$	$c_T = -(52 - 50) = -2$	$c_T = -(48 - 50) = 0$
Short American call	$c_T = -(S_T - X)$	$C_T = -(52 - 50) = -2$	$C_T = -(48 - 50) = 0$
Short European put	$p_T = -(X - S_T)$	$p_T = -(50 - 52) = 0$	$p_T = -(50 - 48) = -2$
Short American put	$P_T = -(X - S_T)$	$P_T = -(50 - 52) = 0$	$P_T = -(50 - 48) = -2$
S_T = Stock price at expiry X= Exercise Price C_T/c_T = Call Price at Expiry P_T/p_T = Put Price at Expiry			
Notes: Results for Short positions are the negative of the corresponding long position			

[22] See http://ir.cboe.com/press-releases/2014/jun-2-2014a.aspx

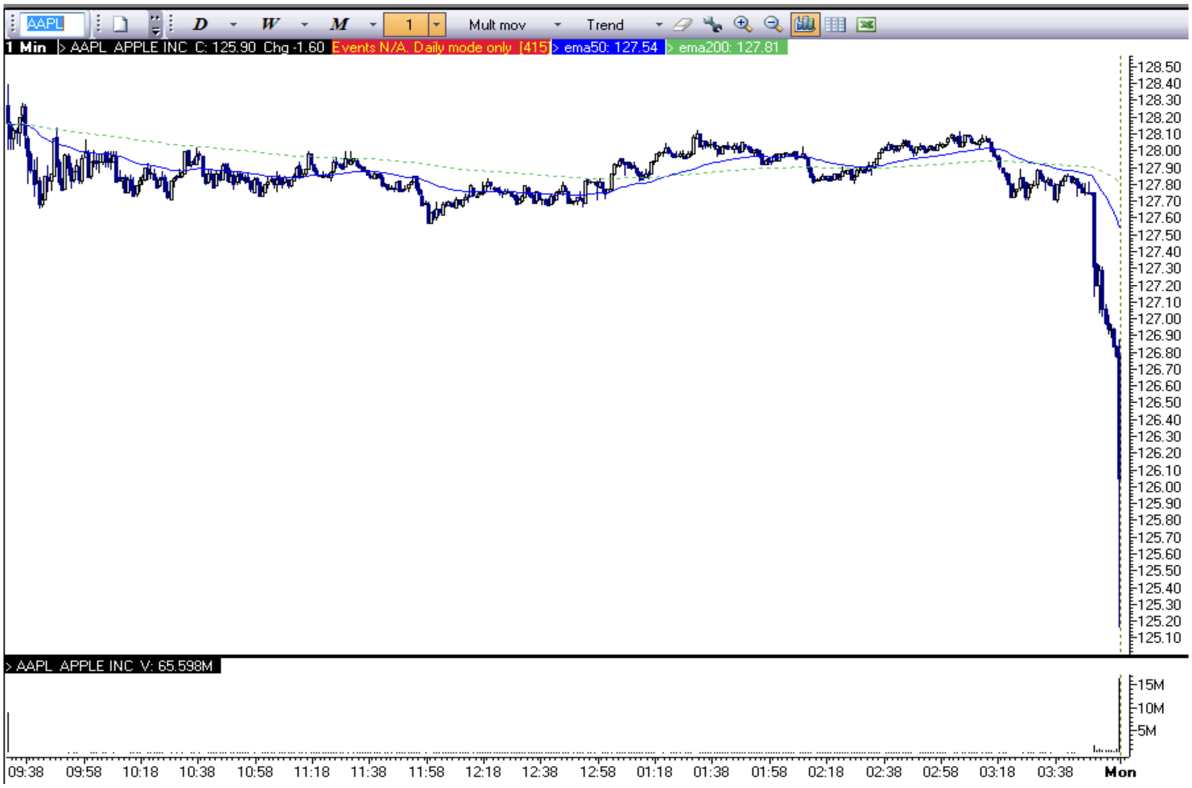
[23] Institute, CFA. 2015. CFA Level II Volume 6 Derivatives and Portfolio Management. Wiley Global Finance, 2014-07-14. VitalBook File.

Appendix 5: Apple Inc. Weekly Call and Put option Open Interest on 03/20/2015 expiration.

Strike	Call	Last	Chg	Bid	Ask	Vlm	OI		Put	Last	Chg	Bid			
15Mar122.00	AAPL1520C122-US	4.85	-0.90	3.85	4.40	260	3010	122.00	AAPL15200122-US	0.01	-0.01	0.0	0.01	896	11875
15Mar123.00	AAPL1520C123-US	3.85	-0.65	2.83	3.40	1K	7581	123.00	AAPL15200123-US	0.01	-0.01	0.0	0.01	2K	13191
15Mar124.00	AAPL1520C124-US	2.23	-1.25	1.84	2.37	3K	11526	124.00	AAPL15200124-US	0.01	-0.01	0.0	0.01	2K	12047
15Mar125.00	AAPL1520C125-US	1.01	-1.53	1.00	1.38	20K	57487	125.00	AAPL15200125-US	0.28	+0.23	0.0	0.01	4K	38350
15Mar126.00	AAPL1520C126-US	0.25	-1.36	0.10	0.48	16K	18499	126.00	AAPL15200126-US	0.02	-0.11	0.01	0.02	11K	14819
15Mar127.00	AAPL1520C127-US	0.01	-0.88	0.02	0.03	37K	25169	127.00	AAPL15200127-US	0.49	+0.13	0.65	1.06	47K	16995
15Mar128.00	AAPL1520C128-US	0.01	-0.36	0.0	0.01	79K	34369	128.00	AAPL15200128-US	1.60	+0.73	1.60	2.04	56K	15005
15Mar129.00	AAPL1520C129-US	0.01	-0.12	0.0	0.01	46K	35622	129.00	AAPL15200129-US	2.21	+0.57	2.61	3.20	8K	8756
15Mar130.00	AAPL1520C130-US	0.01	-0.03	0.0	0.01	20K	72531	130.00	AAPL15200130-US	3.21	+0.68	3.60	4.15	11K	15381
15Mar131.00	AAPL1520C131-US	0.01	-0.01	0.0	0.01	6K	24901	131.00	AAPL15200131-US	4.55	+1.00	4.65	5.10	766	3132
15Mar132.00	AAPL1520C132-US	0.01	--	0.0	0.01	869	17366	132.00	AAPL15200132-US	5.18	+0.93	5.65	6.15	3K	7433

Image ©Yahoo Finance. Available on public domain.

Appendix 6: Apple Inc.03/202015. One minute Chart. 9:30AM – 4:00PM ET



Appendix 7: Percentage change in price in the period between 3:50pm – 4:00pm ET on 03/20/2015.

Security Description	Ticker	% Change
S&P 500 Broad Market Index	S & P	-0.168%
Apple Inc.	**AAPL**	**-2.027%**
Microsoft Corp	MSFT	-0.164%
Google Inc.	GOOG	-0.210%
Facebook Inc.	FB	-0.107%
Amazon Inc.	AMZN	-0.208%
Intel Corp.	INTC	-0.445%
Average decline ex-APPL, ex-S&P 500		**-0.143%**

Source: Thomson Reuters Inc. Used with permission.

Appendix 7: Apple Inc.03/202015. After Market Bid/Ask quotes.

Ticker Symbol		Bid	Ask	Closing	Change	Daily Vlm.	10 Day Avg. Vlm.
PCLN	↓	1176.00	1179.99	1179.17	+2.85	682,720	662,945
PCLN1...	↓	0.0	0.05	0.03	-0.32	225	
AAPL1...	↓	1.76	2.00	1.58	+0.21	9,681	
AAPL	↓	126.85	126.86	126.05	-1.44	63,903,624	59,733,392
GOOGL	↓	564.95	565.19	564.95	+1.28	2,080,341	1,692,196
SPY XD	D	210.77	210.78	210.77	+2.21	131,737,913	137,895,376
NFLX	↓	428.30	428.89	428.30	+3.09	1,902,648	2,318,494
BABA	↓	85.16	85.17	85.16	-0.58	20,466,309	17,943,360
TSLA	↑	198.08	198.30	198.17	+2.52	4,210,920	5,740,633
TSLA1...	↓	1.15	1.21	1.15	-1.15	1,015	
XBI XD	D	233.59	233.61	233.60	-4.13	1,728,114	662,807
UNP	↑	117.47	117.48	117.48	-0.07	3,579,904	3,798,957
JPM	↓	61.77	61.78	61.76	+0.56	15,137,652	15,388,248
MPC	↓	102.27	102.28	102.23	+2.17	2,639,998	2,348,704
WYNN	↑	126.96	132.64	129.98	-0.13	2,336,600	3,529,467
BIDU	↑	212.50	212.70	212.69	-1.88	2,110,981	2,497,400
AMZN1...	↓	0.21	0.24	0.22	-0.53	352	
AMZN	↓	378.55	378.79	378.49	+5.25	3,216,496	2,493,699
GS1517...	↓	0.63	0.73	0.70	-0.55	116	

Note: Final close was $125.90. Screenshot was captured immediately after closing bell, before all the trades cleared the books.

Appendix 8

GLOSSARY[24]

At-the-money
An option in which the underlying value equals the exercise price.

American option
An option that can be exercised at any time until its expiration date.

Beta
A measure of the volatility of a security in comparison to the market as a whole.

Call
An option that gives the holder the right to buy an underlying asset from another party at a fixed price over a specific period of time.

Dark Pools
Private exchanges or forums for trading securities; unlike stock exchanges, dark pools are not accessible by the investing public.

Delta
The relationship between the option price and the underlying price, which reflects the sensitivity of the price of the option to changes in the price of the underlying

Dynamic hedging
A strategy in which a position is hedged by making frequent adjustments to the quantity of the instrument used for hedging in relation to the instrument being hedged.

Equity options
Options on individual stocks; also known as stock options.

Exercise
The process of using an option to buy or sell the underlying. Also called exercising the option.

Exercise price
The fixed price at which an option holder can buy or sell the underlying. Also called strike price, striking price, or strike.

European option
An option that can only be exercised on its expiration date.

[24] Institute, CFA. 2015. CFA Level II Volume 6 Derivatives and Portfolio Management. Wiley Global Finance, 2014-07-14. VitalBook File

Gamma
A numerical measure of how sensitive an option's delta is to a change in the underlying.

Hedging
A general strategy usually thought of as reducing, if not eliminating, risk

In-the-money
Options that, if exercised, would result in the value received being worth more than the payment required to exercise.

Long
The buyer of a derivative contract. Also refers to the position of owning a derivative.

Open Interest
The total number of options and contracts that are not closed or delivered on a particular day.

Option
A financial instrument that gives one party the right, but not the obligation, to buy or sell an underlying asset from or to another party at a fixed price over a specific period of time.

Out-of-the-money
Options that, if exercised, would require the payment of more money than the value received and therefore would not be currently exercised.

Put
An option that gives the holder the right to sell an underlying asset to another party at a fixed price over a specific period of time.

Random walk
A time series in which the value of the series in one period is the value of the series in the previous period plus an unpredictable random error

Short
The seller of a derivative contract. Also refers to the position of being short a derivative.

Straddle
An options strategy with which the investor holds a position in both a call and put with the same strike price and expiration date

Strike price
See exercise price

Theta
The rate at which an option's time value decays.

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