Young Children Representing Numbers:
Implications for Teaching

A Thesis

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Abstract

This qualitative study explored the responses and strategies used by children ages two to five when prompted about numbers and numerals. This study, based on a socio-constructivist framework, was intended to facilitate discussion among early childhood educators and to challenge current mathematical practices in early childhood education.

The children participated in a whole class session and in an individual session which were videotaped. The whole class session involved all children from each age group: a 2- year -old class; a 3- year old class; a 4 year old class, and a 5 year old class). During this session children were asked to count and to locate numerals in the class environment. The individual sessions involved 16 children (two boys and two girls) from each age group. Children responded to three different tasks: a Symbolic Response Task (SRT), a Printing Response Task (PRT), and a Reading Response Task (RRT). The SRT purpose was to observe children's strategies and responses when prompted to use any type of symbolic representation other than oral language. The PRT focused on exploring children's numeric representation after being presented with three boxes that contained different amounts (3, 9, and 14). The RRT purpose was to observe children's reading responses in reference to their own numeric notations.

From a pedagogical perspective, several themes emerged from the observations. Themes such as the role of rote counting, children's one-by one understanding of sets, and children's emotional responses toward reading numerically are discussed. Implications for teaching are suggested based on the children's responses.

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And to my children, Milagros, Ignacio and Francisco, who supported me with their love and warm smiles.

Dedicated to
Javier, Milagros, Ignacio and Francisco

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CHAPTER ONE

Overview

"Uno, dos, tres..." Maria counts while pointing at the bears in the book she always carries with her. Her mom, who is driving, counts as well. They do the same routine every morning while driving to daycare. Maria and her mom count and repeat a Spanish poem together.

That morning, Maria is extremely excited. Maria's class is working on a special project about lighthouses. She is going to show her friends all the lighthouse pictures her family took during the weekend.

Maria walked into the building in a hurry. The classroom is noisy and everybody is busy; some children are building, others are drawing on large posters, and others are getting dressed up.

"Maria", said her teacher, "could you go to the kitchen and ask for five spoons...there are not enough for the snack table."

After she accomplished that, Maria chose to play in the dramatic area. "There are 10 boys in the class today" said Maria while she prints seven tally marks in the white board. Maria is playing teacher with her friends and everybody seems highly enthusiastic. She placed the lighthouse pictures on the bulletin board. Maria remembered each of the places and tried to organize the lighthouses from tallest to shortest.

Later, during work time, Maria received a pencil and a sheet. Her teacher asked her to count the objects on the page and to write the number. Maria looked at the page; she can see a flower, butterflies and something that she is not sure about ("maybe flies?"). Besides each picture there is an empty square that the teacher keeps pointing at.

Maria takes a long time to complete the activity, her eyes look around the classroom and her face is tense. She finally traces a big square around the empty one, passes the sheet quickly to the educator, and runs to the playground.

Introduction

As an early childhood educator (ECE) for more than two decades, I have observed situations like the one previously described in many different early childhood settings. The same attitude was expressed by many young children in both countries, Argentina and Canada, where I have had the opportunity to work. It seemed to me that when young children printed numbers and the printing was not influenced by adults' expectations, the notations and the attitudes were quite different. As happened to Maria, most of these children's responses to formal request were fragile and lacked enthusiasm, as if they did not know anything about mathematics.

My interest in mathematics and in young children's development caused me to question why young children's strategies changed so drastically in routines like the one described above. Why were the responses, including the body language, so different from one situation to the other? Why had Maria printed numeric ideas when she played teacher and was not able to respond when the educator asked her to write numbers?

Early childhood mathematical and cognitive studies have shown that young children use their own notations to represent quantities (Bialystok & Codd 1996; Hughes 1986; Sinclair, Siegrist &Sinclair, 1982 as cited in Bialystok & Codd 1996; Teubal & Dockrell, 2004). This issue was also explored by well-known researchers in the early childhood field such as Constance Kamii, who with other researchers, showed that young children represent numbers in their own way according to their levels of abstraction (Kato, Kamii, & Nagahiro, 2002). After reflecting on these theoretical frameworks, I wondered why we were expecting young children to print conventional numbers (numerals) in early childhood classrooms. Subsequently, I explored more in depth different pedagogical expectations that educators (including myself) had regarding written numbers. It was then

that I had to think deeply about the appropriateness of the activities I proposed while engaging my young students in number representation. I reflected intensively about appropriate pedagogy that responded to and challenged my students' opportunities to represent mathematical ideas. I came to understanding that only by planning appropriate developmental mathematical activities and environments could I keep these young children motivated and engaged.

I collected types of students' numeric written responses for more than ten years. Based on those responses, on my field observations, and on the continuous reflection process in which I engaged, I asked one more question: How could I develop a framework for young children that considered appropriate developmental activities for number printing? How could I support Maria's existing mathematical knowledge and challenge her ways of printing and talking about numbers?

This study is a response to that question, one that I now understand began many years ago.

Rationale

Early childhood education is the primary stage in the lifelong learning continuum, and it provides the foundation for later learning success. It has been shown that the most dynamic period of brain development occurs from birth to age five and that some trajectories in learning and behavior may already be set by the time children enter the school system (Mustard, 2006, 2008, 2009).

Research consistently suggests that quality early childhood experiences affect children's health and well being, lifelong learning, and personal successes (Doherty,

Lero, Goelman, & Tougas, 2000; Friendly, 2008; Kohen, Garth & Hertzman 2006; Prentice, 2006; Williams, 2002). Furthermore, most studies and reports in developed western countries show the positive impact of early childhood education, not only on individuals, but also in societies and governments (Mitchell, Willie, & Carr, 2008; Muller-Kucera & Bauer, 2001; Mustard, 2002; Pierre & Philip, 2005). Consequently, new demands on how to educate future generations in an era of globalization have been raised, and early childhood education has become a long-term investment that most advanced societies find beneficial.

In areas such as mathematics, it has been shown that foundational mathematical knowledge has a tremendous impact on later academic success and on an individual's day-to-day living. The ability to solve problems using mathematical thinking is crucial, for example, for artistic and economic development. Therefore, "Early Childhood Mathematics Education (ECME)" (Ginsburg, Lee & Boyd, 2008, p.3) learning outcomes and skills which are optimally developed during the early years become important domains that should be considered by ECEs. Siegler (2003) identified some of the impacts of poor ECME in elementary students such as little or no understanding of mathematical procedures, lack of connection between concepts and procedures, little or no sense of the relative magnitude of single digit numbers, and relatively poor counting skills.

However, even though many studies have shown how young children develop cognitive and numeric ideas (Bialystok & Codd, 1996; De Loache 1995a, 1995b; Hughes, 1986; Kato, Kamii, Ozaki & Nagahiro 2002; Piaget, 1952, 1962; Piaget & Inhelder, 1971, 1983), little research has been conducted concerning the effectiveness of

specific instructional strategies to teach mathematics to young children (Gifford, 2004). It appeared that in general, most early mathematics teaching has been oriented to school preparation with a tendency to teach content rather than to scaffold mathematical process (Brosseau, 2006; Ginsburg, Sun Lee, & Boyd, 2008).

I believe that many misconceptions about young children's mathematical learning and the lack of pedagogical research about this matter have been guiding early childhood educators to rely, in some cases, on practices that are not developmentally appropriate and that lack clear mathematical intentionality.

One of the contents which generally received considerable attention in ECME is the conventional use of mathematical symbols in their written (numerals) or spoken form (rote counting). These skills involve children using symbols and symbolic notations. In general, a tendency exists to rush children into the use of symbols and in the name of "academic learning," young children are expected to practice and master the use of conventional written numerals (i.e.,1, 25, 100), before entering the school system. Most of the practices involving the use of numerals are presented in worksheets where children repeat the same drills page after page. These mathematical worksheets involve children doing repetitive non-problematic exercises that, according to Van de Walle (2001), give a false appearance of mathematical understanding. According to Hughes (1986), asking young children to print conventional numerals leads to responses where children replace a natural analogic system in representing numbers for one that is more contrived and of no meaning to them. It is what is known as pencil pushing. According to the Plainfield Department of Curriculum "pencil pushing has helped produce generations of people who see mathematics as little else" (p. 24).

In my teaching experience, I have observed that many children are exposed to written conventional numerals from very young ages. Furthermore, many mathematical assessments are based on young children's understanding of conventional numerals. Are these kinds of exercises necessary and acceptable to build foundational mathematical knowledge in our children? Do these types of assessments facilitate our young students to learn the meaning of number and written numbers? Do these assessments inform educators about what children can really do?

This study is an invitation to reflect about the pedagogical principles that underline those pencil pushing practices as well as to reflect about the conceptual frameworks that underlie current early childhood mathematical practices.

Significance of the study

Working in early childhood education has led me to experience how, in most situations, mathematics is viewed as an isolated academic "area of learning," with little integration with other areas of child development. Furthermore, mathematical activities are often very poorly connected with real life situations that in general are more meaningful for young children.

According to Anderson, Anderson and Shapiro (2005) "mathematics is a social practice that is embedded and influenced by particular social and cultural practices" (p. 8). Socio-cultural ideas are supported by previous studies (Cobb, 1994; Rogoff & Chavajay, 1995; Vygotsky 1978) who also maintained that children are participants in socio-cultural practices and that learning happens as the individual interact with others.

This implies that mathematical education should facilitate and support the development of cultural connections within children's environments.

Mathematics cannot be learned in isolation. Comprehension and understanding of foundational mathematical concepts can be achieved by young children if there is a developmentally appropriate curriculum that gives them the opportunities to experience numbers, quantities, and numerals many times and in many different ways while interacting with their environments. According to Ginsburg and Golbeck (2004), mathematics curricula for young children should "focus not only on content, but also on process, on mathematical *thinking*. Indeed, one might say that mathematical thinking is the content" (p. 197).

Perhaps because of the overall tendency to isolate mathematics, numerals are seen as an important achievement that needs to be produced in isolation rather than as a cultural tool that develops progressively and requires understanding of specific numeric outcomes such as quantity. It appears that the timeframe for producing conventional numerals is shortening, and year after year, young children are expected to print numbers earlier in age. Sadly, this activity is completed by many young children with little engagement and enthusiasm and most of the time, with no opportunity to explore and comprehend the intention of the number printing.

Scaffolding children through foundational mathematical concepts, including number printing, should be supported by: 1) ECEs' understanding of mathematical processes, and 1) ECEs' understanding of children's knowledge. Kamii (1981) referred to this as "scientific training of teachers" (p. 5). According to Kamii, teachers need to base their decisions on "the scientific study of children." Furthermore, Kamii states, "teachers

today generally base their practice on their common sense and intuition about what feels right rather than scientific knowledge of how children develop" (p. 5).

This belief is also shared by Carpenter, Fennema, and Frankee (1996) who suggest that the only way to teach mathematics meaningfully is through an understanding of the sequence of strategies children use when solving or being confronted with a mathematical situation. Educators' mathematical knowledge and the educators' knowledge of how young children "think math" (Franke & Kazemi, 2001) are key components, according to these authors, of developmentally appropriate mathematical practices. Based on this approach and with the intention of exploring number printing, my research demonstrates that it is critical to document more precisely how children acquire and develop the ability to represent numbers. The purpose of this qualitative study is to start the process of creating a pedagogical framework based on the existing literature and on children's responses that could demonstrate when it is appropriate to introduce written numeric representations, as well as what indicators should be considered by educators when engaging young children in this activity.

This study also sets out a context for discussion about developmental mathematical practices through the early years and challenges us to reflect about the ways pencil pushing happens in isolation during a time when individuals are discovering the world.

Terminology

To clarify the terminology used throughout the study, a brief list of terms is included below:

Number: is an abstract entity of knowledge. My research study is based on Piaget's (1952) ideas about number construction. Piaget maintains that number is constructed in close relation with an understanding of a system of inclusions (that relates to classes, for example, utensils) and its relation with a system of order (for example, short spoon, medium fork, tall knife). For Piaget number is at the same time both class and order. Understanding numbers implies, for example, understanding that 6 spoons include 1, 2,3,4,5 and 6 spoons and that those spoons are a class that is included in a larger class (eating utensils). For Piaget (1952), this understanding is constructed by individuals through different developmental stages.

Cardinal meaning: refers to the value of the number in terms of quantity. It responds to the question "how many." Cardinality, according to Bermejo (1996), is a way of quantifying all the items of a set. Cardinality refers to the classificatory meaning of natural numbers (for example, all classes containing "3" without reference to whether they are spoons, forks, or knives)

Numerals: are the conventional written symbols used to represent a number, for example, "6," "100," or "35" (Kato, Kamii, Ozaki, & Nagahiro, 2002). They are the conventional written symbols that communicate the total amount in a set.

Numeric notations: are the children's own written ways to represent numbers (Teubal, & Dockrell, 2005), for example, the use of scribbles and tally marks. Accuracy and quality of the representation is sometimes not contemplated in these kinds of printing representations. For example, a child could represent "five" by printing four tally marks,

or a perfectly drawn "3" (Teubal & Dockrell, 2005).

Symbolic responses: refer to any form of response used by children that implies the use of a symbol to represent a quantity, for example words, use of manipulatives, or printing (DeLoache, 2004).

Types of responses: refer to the categories of responses children give in each task, for example, representing quantities through drawing objects or using a one-to-one reading response.

Strategies: are the different approaches and/or meaningful actions that a child does to accomplish or make sense while counting, printing, or reading numbers and numerals (for example, oral counting, finger counting or using visual cues). The study assumes that sometimes body language could express strategies used by children. Therefore, the study aims to describe children's body language as they respond to the different tasks.

Need for the Research

The current body of literature provides an extensive range of research describing how the domains involved in number representation, symbolic domain, and numeric domain develop gradually during the first years of life. The literature review also addresses how important these domains are for children's cognitive development.

If symbolic and numeric developments are emerging during the first five years of life, why are young children being rushed to write numbers? Why are numerals expected to be achieved and used by young children before entering school?

The lack of pedagogical research about *when* and *how* to introduce young children to number printing have been the guiding questions for this study. I believe that

documenting and understanding how numeric notations evolve have important consequences for understanding cognitive development, and also should have implications for ECME meaningful teaching. Because of the value of documenting the developmental process that children follow when printing numbers, a qualitative approach was a natural choice for my methodology. The exploration addresses three main components of number printing that I developed based on previous studies and on my own understanding of the issue: symbolic response, written response, and reading response. This study explored which form of symbolic responses, other than spoken words (for example, the use of manipulatives or printing) children produce when asked about numbers and numerals. Children's written responses were examined through the exhaustive documentation of young children's numeric notation processes. Children's reading responses are described based on children's understanding of their own numeric notations. An in-depth exploration of strategies used by children while printing numeric notations is also a key component that was analyzed.

It is my aim that this study will contribute to early childhood education and to expand previous mathematical and cognitive studies about the developmental process of number printing. As an ECE, it is my intention that my study will encourage and challenge early childhood mathematical practices, especially those regarding number printing. Reflections about the way we do math in the early years, in particular reflecting about the rationale of certain practices, could lead ECEs to the valuable consideration of young children's thinking mathematical processes.

Research questions

In a series of studies, Byalistock and Codd (1996) explored the role and function children give to numerals and to non-conventional numeric forms. The purpose of those studies was to assess how children understand the cardinal meaning of their own notations. Partial adaptations from Byalistock and Codd were considered when I developed a series of research sessions and the questions for the specific protocols of this study. My teaching experience while working in the early childhood field, my special interest in mathematics and my continuous involvement with mathematical training have also had an impact on the way I organized and developed the research sessions and the research questions for this study.

My study explored these questions:

1. What symbolic responses do young children chose when prompted to represent numbers?
2. What strategies do young children use when prompted to represent numbers?
3. What numeric notations do young children use when asked to print numbers?
4. What are the developmental trends of young children's numeric notations?
5. What strategies do children use while printing numeric notations?
6. What meaning do these notations have for the children?
7. How could ECEs support children's developmental trends and sequence of strategies used in number representation?

The sites of my research sessions were two Early Learning Centres (ELCs) from Prince Edward Island that employed ECE certified staff. The participants were children

from both ELCs ranging in age from 2 to 5 years old. My sources of data included videotaping, observational records, children's notations, and my researcher's journal.

Young children's active role in the fascinating journey of learning has been highlighted by the findings provided. I concur with Brousseau's (2006) statement that "Mathematics is produced by mathematical activity" (p. 6). However, I will add to this statement that mathematical activity needs to be developmentally appropriate to become meaningful for our young students. This is the only way to ensure that children like Maria become and will continue to be engaged and interested in Mathematics.

CHAPTER TWO: LITERATURE REVIEW

This chapter presents previous studies' frameworks and ideas. The main themes that I identified for literature exploration are: constructivism and social constructivism, early childhood mathematics education, the use of symbols, number representation, children's knowledge, and mathematics linkages with pedagogy.

Introduction

Over the last ten years, cognitive and neurological research has provided an important body of knowledge that is changing the value society places on the first five years of human development (Bruer & Mustard, 2002, McCain, Mustard & Shanker, 2007; Mustard, 2006, 2008, 2009). According to Mustard, the most dynamic period of brain development occurs from birth to age five and some trajectories in learning and behavior may already be set by the time children enter the school system.

These new values which are supported by the science of early brain development are changing society's understanding about children's minds. Better early childhood education appears to be a long-term investment that most advanced societies are willing to make. Consequently, the field of early childhood education has begun to place renewed emphasis on the need for a different teaching approach.

In areas like mathematics, it is clear that foundational mathematical skills which are optimally developed during the early years, facilitate a person's later willingness to learn and believe in the value of mathematics for everyday living and problem solving. These ideas are supported by many studies that have shown how children develop cognitive and numeric ideas (Bialystok, 2000; Bialystok & Codd, 1996; Bialystok & Martin 2003; Kamii, 2000; Kato, Kamii, Ozaki, & Nagahiro 2002; Piaget, 1945, 1962;

Piaget & Inhelder, 1971). However, there is a lack of pedagogical understanding about which are the best practices to support children's numeric development and the contents and outcomes of ECME are still unclear. Many early mathematical domains such as number sense, operations, and number printing, are still under debate. Overall, many inconsistencies exist in the early childhood field concerning number printing.

As a believer in mathematics and mathematics teaching, I have always been interested and curious about the tremendous gap that tends to exist among mathematics, cognitive theories, and practical classroom applications for young children. I truly support the value of creating the best context for my students based on their individual development. Young children's engagement and curiosity in mathematics can only grow and develop if there is a clear understanding of how children "think math" (Carpenter, Fenemma, Frankee, Levi, & Empson, 1999). Personally, I understand that by knowing how children's mathematical ideas develop educators could better provide quality teachable moments that will enrich play and organize intentional teaching.

This study will explore how young children "think" numbers and number printing and the areas explored provide the rationale for this study. The literature review considers research from other studies as well as important theoretical concepts related to the early childhood field and to ECME. I conclude the literature review with a summary of the current literature about Cognitive Guided Instruction (CGI), an approach which frames the methodology process used in the study.

Theoretical framework

Supported by a constructivist framework, this study explores the process of knowledge construction that young children appear to follow when developing the idea of numbers and numerals. As a constructivist educator and researcher in this current inquiry, I also experienced a tremendous personal growth. This process of constructivism is described by Schwandt (2000) as an epistemological "understanding of understanding where a knower (the inquirer as subject) gains knowledge about an object (the meaning of human action)" (p.194). In other words, the researcher involves herself in a dialectical process, where understanding of others' responses and his/her construction of meaning are inseparable (Lincoln & Guba, 2000).

Constructivists understand that knowledge is not passive, rather humans construct it or make it; "We invent concepts, models, and schemes to make sense of experience and we continually test and modify these constructions in the light of new experience" (Schwandt , 2000, p. 197). These theoretical arguments about how humans construct knowledge have impacted educational research from different historical periods. As Gordon (2009) expressed, authors such as Plato, Locke, Kant, Rosseau, Pestalozzi, Piaget, Vygotsky, Dewey, Freud and Foucault, have been impacted by different ways of understanding constructivism.

My study is supported by the idea that this construction of knowledge does not happen in isolation but within culturally shared understandings and practices. From a socio- constructivist perspective, mathematical learning is a process of individual construction and a process of enculturation into social mathematical practices (Cobb, 1994). Therefore, mathematical learning is considered as a process of active construction that occurs when individuals engage in mathematical practices with others.

Symbols appear to play a particular role within the social constructivist framework. According to Cobb (1994) symbols carry a shared social meaning that individuals have to construct. Particular attention is paid in this study to the way that written numeric symbols are constructed by children to communicate social mathematical meaning.

Early Childhood Mathematics Education (ECME)

Mathematics in the early childhood field has been widely influenced by the ideas of the constructivist school. For Kamii (1985), constructivism is "the theory according to which the child builds his (sic) own knowledge from the inside, through his (sic.) own mental activity, in interaction with the environment" (p. 6). Constructivists believe that knowledge is not directly transmitted from the teacher, but that teachers can facilitate knowledge acquisition.

The vision of the child as an active learner, capable of understanding abstract concepts through exploration and manipulation, grew tremendously in the early childhood field, especially in opposition to the passiveness often related to the traditional school system. These ideas were mainly supported by Piaget's theory and the three kinds of knowledge he described: physical, social, and logico-mathematical (Piaget, 1953, 1962). Each kind of knowledge is related to the others, and according to Piaget and Inhelder (1971, 1983), empirical and constructivist abstraction facilitates their acquisition. Empirical abstraction supports the individual's focus on different properties of the objects (i.e., colour, size, and weight) while constructive abstraction involves mental relationships that the individual makes among objects (i.e., two, the same, and different).

Under the constructivist approach, play and hands-on activities became the strategy mainly used to facilitate young children's learning. Hands-on activities and manipulation were related to free exploration and oriented to facilitate empirical abstraction. However, misconceptions of abstraction led those in the field to believe that young children could learn abstract mathematical ideas by just touching objects (Kamii Lewis & Kirkland, 2001; Williams & Kamii, 1986). Consequently, these kinds of activities lacked clear mathematical intentionality, and the role of the educator was limited to providing objects and observing (Kamii Lewis & Kirkland, 2001; Williams & Kamii, 1986)

The idea of exploration, mostly related to play, is still very important in the field. The environment and the presence of various manipulatives are considered key components of mathematical learning. However, the variety of manipulatives and free exploration do not necessarily facilitate or guarantee learning (Williams & Kamii, 1986). When talking about the value of play, Kamii (2005) suggests: "Play has always been valued in early childhood education; it is important for educators to know precisely *why* a playful activity is educational and what the teacher can do to maximize its value" (p.382).

The latest mathematical research emphasizes that the only way that manipulation can become meaningful in a rich environment is if it is guided through reflection and problem solving (Brosseau, 2006; Van de Walle, 2001). In other words, empirical abstraction should be supported by a challenging environment where constructive abstraction is encouraged and challenged. Using this approach, the educator's mathematical knowledge and the educator's knowledge of how young children "think math" (Franke & Kazemi, 2001) become key components of developmentally appropriate

mathematical practices.

The Use of Symbols

It is interesting to observe that ECME has primarily related the use of symbols to what is known in the field as "academic learning". Mathematics has often been lumped with literacy under this label (Sun Lee & Ginsburg, 2007), and many early mathematics outcomes that involve the use of symbols have been oriented to school preparation.

In the case of mathematics, it appears that "academic learning" was mainly related to the use of mathematical symbols in their written or spoken form. Thus, to facilitate the development of numerals (the conventional printing for numbers), intensive practice on the paper was required in many early childhood settings. Consequently, "ready-made" activities (Sun Lee & Ginsburg, 2007, p. 135), also called worksheets, were presented to the child from very young ages. In general, educators who expressed a strong commitment to academic education were the ones who approved the use of these kinds of activities (Sun Lee & Ginsburg, 2007). The appropriate use and understanding of numerals (1, 8, 15, 100, and so on) still receives considerable attention, formally and informally, during the early education years. This involves young children using symbols and symbolic notations. It is interesting to observe that even though play and hands-on activities are key components of ECME, a tendency exists to rush children into the use of symbols.

When trying to define the term symbol there is a language inconsistency. Different theorists used terms such as "symbol", "icon", and "sign" (DeLoache, 2004). Vytgotsky's work (1978) explored the role of symbols in children's cognitive development. In his studies, Vytgotsky differentiates between a first and a second order

symbolic system. For him, first order symbols convey a pictorial message and it is in these cases where the symbol can be decoded just by looking at it (i.e., a picture of a cat resembles a cat). The second order symbolic system uses a graphic form that represents an abstract unit of meaning, like a phoneme or a quantity. It is in these situations where even by looking at the form, the conventional meaning will not be revealed (i.e., "100", "A", "casita"). According to Vytgotsky, understanding the meaning of conventional notations is more difficult than understanding the meaning of pictures. For DeLoache (2004), the child needs to understand the social intention to communicate that different symbolic systems have. For example, the child needs to understand what the printed notations "12", "MOM" or "58" mean. This idea implies that the meaning of symbols is grounded in social patterns and patterns of communication.

According to DeLoache (2004), "symbols are a characteristic of humans. A vital function of symbols is to enable humans to acquire information without direct experience. Our vast store of cultural knowledge exists because we can learn through symbolic representation" (p.68). For this researcher, intention and communication are at the basis of symbols, and children need to figure out how people intend symbols to be interpreted.

For DeLoache (2004), symbols represent, are general, and are intentional. Symbols represent because they denote; they are about something. According to DeLoache, anything can be used to represent: numbers, words, sounds, fingers, blocks, maps, and many other possibilities. She states that the intentionality of symbols is grounded on social contexts; therefore, there has to be a person who intends to represent. The same

approach was suggested by Piaget (1953; 1962; 1971) when he explained that it is people who represent, not the symbols.

For Hobson (2000), intentionality is the foundation of symbols: "intention is at the heart of symbolization" (p. 2). He describes this intention as an intention-to-refer and intention-to-mean where there is a mental relationship between the symbol and what, in fact, it represents. This mental relationship between the symbols and what they signify is the unique dual nature of symbols (Uttal, Scudder, & DeLoache, 1997). Based on her theoretical model DeLoache (1995a; 1995b) calls this relationship between a symbol and what it represents, dual representation. She explains that dual representation supposes a representational insight and the capacity to represent simultaneously the idea of "5" and its abstract relation, the word "five", "5"or, "IIIII." This capability considers the capacity to mentally represent the concrete object itself and its abstract relation to what it stands for. A clear example is provided by MacConnell and Daeheler (2004). They state that a child is capable of dual representation when he/she is capable of perceiving a model train as both a toy and as a representation for an actual train. The capability to use dual representation is discussed by MacDonnell and Daehelr (2004) as symbolic insight. For Piaget (1952, 1962) the individual has to "see" the idea mentally in order to be able to represent it.

Most researchers agree that dual representation is an important cognitive milestone that children need to achieve in order to understand symbols. An understanding of this abstract relation among symbols and what they represent is quite difficult to achieve at young ages (DeLoache, 1995a, 1995b; DeLoache & Burns, 1994; DeLoache & Marzolf, 1992; DeLoache, Miller & Rosengren, 1997). During the first periods of life, children

understand symbols mainly as objects and it seems that the younger the child, the more inclined the child is to do so (DeLoache, 2004). Some studies (Byalistok, 2000; Byalistok & Codd, 1996; DeLoache, 2004; DeLoache & Burns, 1994; DeLoache, Miller & Rosengren, 1997; Huttenlocher, Vasilyeva, Newcombe, & Duffy 2008; Klein, & Bisanz, 2000) are showing that this abstract relationship among objects and symbols is something that children learn progressively during the first years of life.

Number Representation

According to McCloskey (1992), it is important to distinguish between numbers and numerals. The "number" is the abstract entity, the domain of knowledge. A numeral is defined as the conventional written form for numbers (i.e., "1", "5", "77"). Some researchers (Kamii, 1986; Lehere, Schauble, Carpenter, & Penner, 2000, as cited in Teubal & Dockrell, 2005) maintain that conceptual mathematical knowledge should precede children's exposure to the symbolic domains and/or the use of numerals. Others, (Bowers, 2000; Miller, 1996; Sfard, 2000, as cited in Teubal & Dockrell, 2005) argue that a parallel development exists between the conceptual and numeric domain.

This study is based on the conception that young children should construct a solid foundation of mathematical concepts before being rushed or pushed to use symbols. Without a conceptual understanding of the number domain, numerals are not relevant for the child. This study is based on the conception that rushing young children into the use of numerals does not facilitate a clear understanding about the abstract nature of numbers, and could have a negative impact on later mathematical learning.

A numeral is an abstract symbol that stands for a certain quantity for example, "6" apples, "10" fingers, "20" crayons. To be meaningful, the numeral has to be linked to a

number in terms of cardinality. Cardinality is a complex issue in number development and it takes time to develop (Bermejo, 1996; Bermejo; Fluck, Linnel & Holgate, 2005; Morales, & Garcia de Osuna, 2004; Zhou & Wang, 2004). While ordinality refers to the order of natural numbers (i.e., 1^{st}, 4^{th}, 10^{th}), cardinality refers to the classificatory meaning of natural numbers (i.e., 1 refers to all classes containing 1; 6 refers to all classes containing 6 things).There has been much debate about cardinal development; however, it is important to explain that cardinal understanding is at the center of the number domain. Cardinality and ordinality are both aspects of number concept (Kingma, & Koops, 1981; Piaget & Inhelder ,1983). Cardinality, according to Bermejo "is a way to quantify the items in a set" (1996, p. 263). Gelman and Gallistel (1978, as cited in Fluck, Linnell, & Holgate, 2005), identified cardinality as one of the counting principles. According to these researchers, the cardinal principle implies the largest number word in a set. In other words, to understand cardinality children need to understand that "6" includes "1," "2," "3," "4," "5," and "6". Additionally, children need to understand that "6", is always "6", without mattering if the numeral represents 6 balls, cars, or dolls. The last number's word is the spoken symbol used to count the items in a set (i.e., "seven," "fifty," and "twelve"). By extension, it could be suggested that numerals are the written symbols to communicate the total amount in a set (i.e.,"7," 50," and "12"). In other words, both the oral word and the written symbol serve the same function which is to represent the total amount of the items in a set. To use written numerals meaningfully, the children need to understand the cardinal value of the symbol. Therefore, dual representation in the case of any written numeral implies for the child to be able to relate mentally the numeric and the symbolic domains.

Children's knowledge

Young children are surrounded by a world of symbols. From a young age, children need to deal simultaneously with a variety of symbols such as letters, numbers, signs and pictures. In order to participate in their own societies, children are expected to learn about different symbol systems and how they function. Today, as DeLoache (2004) states, "children must learn to use more varieties of symbols than ever before" (p.66).

The exposure to this tremendous variety of symbols happens even before children enter the formal school system. During the early years, the quality of this exposure will facilitate, or not, children's understanding of these symbols (i.e., letters, numbers and what do they mean (what social intention they have) in their social context.

Children's use of symbols progresses from a general symbolic ability (DeLoache, 2004) to a more conventional and intentional ability. For example, the use of sounds to label objects at 13 months changes for a preference to use words at around 18 months (DeLoache, 2004); the use of scribbles while drawing at 2 years of age changes to intentional drawing by 5 years of age. As stated previously, different studies (DeLoache, 2004; DeLoache & Burns 1994; DeLoache & Marzolf, 199; DeLoache, Peralta de Mendoza, & Anderson, 1999) have shown that the mental relation among the symbols and what in fact they represent is not completely developed at young ages and that young children understand symbols as objects themselves.

From very young ages, children become familiar with small numbers and the symbols that are used to represent them: candles on a cake, songs, finger plays, row counting, and so on. However, familiarity with some of these symbols does not guarantee that young children understand the mental relationship for which those symbols stand.

There is a general tendency to believe that children have mastered the numerical domain because they are familiar with some structures of the domain (for example, counting up 20 or playing with statements such as "1, 2, 3...GO!"). Most of this familiarity is associated with memorized oral routines (Byalistock, 2000).

Before using numerals, children use their own creative ways to represent numbers. This study will consider children's different ways of printing numbers as numeric notations. Some studies (Bialystok & Codd, 1996; Hughes, 1986; Kato, Kamii & Nagahiro, 2002) have shown that even when young children knew how to write numbers, the children did not use this knowledge to solve mathematical problem situations and instead used their own forms of numeric notations. Why? Researchers agree that the numerals as symbols did not convey any meaning for these children. Therefore, the numeric symbols (numerals) as a resource to communicate and gather mathematical information were not fully understood. This framework could also be related to Piaget's (1983) idea about children representing their own thinking about reality.

Teubal and Dockrell (2005) suggest there is a significant age effect; therefore, with age children produce more accurate representations and use digits more often. Many studies have explored the type of numeric notations children use (Bilaystock & Codd 1996; Hughes 1986; Kato, Kamii, Ozaki, & Nagahiro, 2002; Sinclair, Siegris, & Sinclair, 1983, as cited in Bialystok & Codd 1996). Even though these studies named the children's levels of representation differently, the development trends are very similar. The studies show that a pattern of development exists for numeric notations that progresses from idiosyncratic notations into more analogical representation, and finally, into the use of conventional numerals.

Hughes (1986) worked with young children and categorized their responses into four levels of representation:

- idiosyncratic: the representations are ambiguous and do not relate to quantity; for example, the use of scribbles, drawings, or pretend writing.
- pictographic: the responses that represent the quantity as well as the characteristics of the objects, such as shape, position, colour or size.
- iconic: consist of the use of recording marks that are in one-to-one correspondence with the objects; for example printing five tally marks or notations such as "12345" to represent "5."
- symbol responses: consist of the use of conventional numerals or written number words; for example printing "3" or "three" to represent three objects.

Sinclair, Siegris, & Sinclair (1983, as cited in Teubal & Dockrell, 2005), in their study of children ages 3 to 6, distinguished the following types of numeric representation:

- global representation of quantity: the representations include a vague idea of "a bunch"; "many." For example, the child will include multiple lines IIIIII to represent three crayons.
- representation of the object-kind: the representation focuses on the qualitative aspect of the set; for example, the child will represent a picture of a crayon, or a letter "C" for three crayons.
- one-to-one correspondence with symbols: this is the first notation type in which numerical ideas makes an appearance; for example, some children invent symbols or use three conventional letters "THR" for three crayons.

- one-to-one correspondence with numerals: It is suggested that children need to use numerals and need to represent each object separately; for example, the child could use "123" or "333" to represent three balls. According to Kamii (2002), these kinds of representations, as well as the type 3 representations, indicate that the child still attends to each object instead of to the total quantity.
- cardinal value alone: the child will use conventional numerals; for example "3" for three crayons.
- cardinal value and object-kind: these notations represent the child's ability to think simultaneously about numeric quantity and object type; for example, the child will use the conventional numeral "3" and an invented spelling for crayons, "3 CRNS."

Sinclair's study showed that these types of representations were related to age groups. The study also showed that half of the children used more than one type of representation.

For Teubal and Dockrell (2005), children's numeric notations could be considered in terms of quality and accuracy. For example, if the child uses a perfectly executed "3," to represent the number "6,"the quality is good, but the accuracy is not. Accuracy and quality are important elements in the development of number notations. In their study about numeric notations, Teubal and Dockrell (2005) concluded that children's internal representation of numbers influence their notational representations.

The construction of number concept involves, as explained previously, an understanding of cardinality in terms of class inclusion and conservation. Many studies show that the principle of cardinality is incomplete for children under 5 or 6 years old

(Fuson, 1992, 1988; Piaget & Inhelder, 1983, 1971; Wynn, 1992, 1990). For Piaget (1962, 1945), cardinality is achieved during the operational stage, at around seven years of age. Prior to this stage, children at a pre-operational stage (three to five years old) represent numbers in their own way, according to their levels of abstraction.

According to Kato, Kamii, and Nagahiro (2002), the level of representation is related to the level of abstraction. These researchers assessed children's level of abstraction through conservation tasks and related the levels to levels of number representation. Their findings are very interesting. They found that the relationship between levels of abstraction and of number representation was significant. The study showed that most children (72%) were at the same level on both tasks: (a) conservation and (b) number representation. In conclusion, Kato, Kamii, and Nagahiro's (2002) study shows that if a child's level of abstraction is not developed enough to understand conservation, it is not developed enough to make representations of numbers as units, that is, by using conventional numerals.

Linking mathematics with pedagogy

Guy Brosseau's research conducted over the last thirty years has produced an important understanding about children's knowledge about math, but also supports teachers in how to teach math. According to Brosseau (2006), traditional didactics was guided by literature and not by the mathematical activity. He suggests that most of this literature has naive interpretations about math and prepares "readers and commentators rather than authors of math" (Brosseau, 2006, p.8). In a plenary session presented in 2006, Brosseau redefined math by explaining that the only way to produce mathematical knowledge and learn math is by producing math. This framework addressed not only the

children, but also the teachers as learners. According to Brosseau (2006), only by producing math, can teachers choose and organize math situations that are appropriate for their students. This is what he calls "didactic engineering" (Brosseau, 2006, p.11).

The same approach that considers teachers as learners of math is used by a professional development program developed in 1992 called Cognitively Guided Instruction (CGI) (Carpenter & Fenemma, 1992). CGI focuses on the development of a child mathematical thinking and is intended to help elementary teachers to "construct conceptual maps of the development of children's mathematical thinking in specific content domains" (Carpenter, Fennema, & Franke, 1996, p.4).

Videotapes of students solving problems and of classroom discussions are the two main strategies used by the program. As the teachers view the video, they are told to focus on the thinking of the students. The main goal of watching the video and sharing ideas about it is to have the teachers construct an understanding of students' capabilities around basic mathematical concepts. Carpenter, Fenemma, Frankee, Levi, and Empson (2000) observed that by understanding the sequence of strategies students used, teachers were able to understand the mathematical domains associated with a particular problem. Furthermore, the study shows that this understanding changed teachers' practices and beliefs about mathematics. Consequently, teachers' expectations of their students changed.

My study adapted the CGI conceptual framework in videotape sessions with 2, 3, 4, and 5 year olds. Building on that framework with partial adaptations from Byalistock and Codd (1996), a series of research sessions observing specific protocols for children's number representation were developed. My goal was to develop a "conceptual map" of

how children construct and understand number printing and to observe the main concepts and strategies associated with this development.

Summary

The major focus of this literature review was to explore the theoretical ideas and research frameworks that refer to number printing during the early years. It is clearly suggested by the literature review that ECME cannot deny the mathematical practice of printing numbers and it should provide appropriate contexts for children to experience it.

Research evidence supports the idea that the relation among numeric and symbolic domains is weak before the ages of 2 to 6. Young children print numbers or use their own numeric notations without understanding why these symbols provide information about quantity. By observing how children develop their own ways to print numbers and what strategies they use while doing it, this study aspires to develop a better understanding of how and when number printing could become appropriate and valuable for ECME. The results of this study could also support creating better links between mathematical theory and practice.

In this study, children's strategies and children's notations became key components of my exploration. The way they responded orally, and the body language they used while counting and printing, became crucial to understanding their mathematical thinking from an educator's perspective.

CHAPTER THREE: RESEARCH METHODOLOGY

Research Design

This chapter presents the research approach taken in this study. I have provided details regarding the research sites, how participants were selected, and various sources of data. Why these sources were chosen and how they were used is crucial for the reader's understanding of how children's observations were documented. Analysis and management of the collected data is described in this chapter as well. Special consideration is given to my researcher's role which includes a description of my professional background, as well as my pedagogical interests to help the reader understand my personal involvement with this study.

A constructivist approach was chosen to conduct this study. The reason for this choice flows from the constructivist view of learning I developed through my teaching experience. According to the constructivist framework, knowledge consists of constructions and these constructions represent human efforts to make sense of the world (Kincheloe, 1991, as cited in Lincoln and Guba, 2000; Schwandt, 2000). This theory implies that knowledge is not passive; in other words, individuals do not perceive knowledge but construct it. Schwandt (2000) states, "if we believe in the construction of knowledge, we believe that the mind is active" (p.197). As an educator, I view young children as active individuals who construct mathematical knowledge progressively. I understand that teaching mathematical meaning at young ages is about facilitating the emergence of these constructions, rather than teaching final products. For this reason, this study explores and describes children's trajectories of number representation, aiming to understand the complexity of this process in order to support mathematical practices.

This study explored the participants' sequence of constructions regarding numbers and numerals. The descriptions and analysis of the children's responses are supported by previous studies (Byalistok, 2000; Byalistok & Codd, 1996, 2000; DeLoache, 2004; DeLoache 1995a; 1995b; Kato, Kamii, Ozaki, & Nagahiro 2000; Piaget, 1952, 1962 Piaget & Inhelder 197; 1983). In addition, I also focused on making sense of the participants' responses. The children's responses and strategies became the main focus of my reflections. My constructivist researcher's role required me to continuously engage in the process of reflecting, modifying, adapting, and expanding my own understanding and knowledge of the children's numeric development.

The framework that supported my observations and descriptions of children's trajectories is based on Carpenter, Fennema, and Franke's Cognitive Guided Instruction (CGI) approach (1996). CGI supported the development of a clearer understanding by elementary teachers of children's mathematical thinking. During different sessions, teachers were presented with a series of videotapes where they observed and discussed how children solved mathematical problems. Based on this framework, during my research sessions, I observed children's mathematical responses. These responses were videotaped; these videotapes allowed me to engage in continuous observation and analysis throughout the study's data collection process. Observation and documentation of the developmental trends and the sequence of strategies used by children aged two to five, as they were engaged in representing, printing, and reading numbers, became the center of this study. Following Carpenter, Fennema, and Franke's approach (1996), reviewing and reflecting on the videotape facilitated my own understanding of the

mathematical domains and strategies associated with number and number representation at young ages.

Building on the CGI framework, and with the purpose of creating an in-depth description of children's responses while counting and looking for written numerals, whole class research sessions were videotaped in eight different ELC classes. In addition, observations based on a framework developed by Byalistock and Codd (1996) were conducted in individual sessions with children. In these sessions, the responses made and the strategies used by 16 children while representing, printing, and, reading numbers were videotaped as well.

The purpose of the Byalistock and Codd's (1996) study was to assess the understanding of the cardinal meaning children (ages 3 to 5) have about their own written numeric notations. After a screening test, the participants on that study were presented with three boxes. Each box contained less than 10 different toy animals. The participants and the researcher counted the items for each box. After that, the children were asked to write how many items each box contained and to place the notation on the top of each box. Later on, (20 minutes) the participants were asked to say how many were in each box by looking at their own notations. The children's reading levels were also assessed by showing children three cards: one with a numeral, one with dots and one with a picture that resembled one of the animals that were in the box.

Bialystok's and Codd's quantitative results came to well documented conclusions about cardinality and children's symbolic understanding. However, this research study has a different purpose, because it aims to explore the pedagogical consequences of children's responses in their construction of cardinality.

The findings of my study are rooted in the analysis of the children's responses and in the developmental sequence these responses seemed to follow. The study also gave special consideration to the strategies young children used when asked about numbers and numerals. Based on these observations, the study lead to the identification of some principles about numbers and numerals for the early stage of an Early Childhood Mathematical Education pedagogical framework. The edited video has the potential to provide ECEs with new knowledge that facilitates their own construction processes about how children understand numbers and numerals.

Research Site and Population Selection

The research sites for this study were two Early Learning Centres (ELCs) in Prince Edward Island. Children aged 2 to 5 from both ELCs were invited to participate. Two key characteristics of the ELCs were identified prior to participation: (1) the Centre was licensed, and (2) the Centre's staff was certified under the Provincial Government Regulatory Board. These characteristics were important, since the ECEs had to rely upon their professional experience to select the children who participated in the individual sessions. This selection process was guided by the ECE's expertise and knowledge of children's age appropriate cognitive, socio-emotional, and physical development.

The ELCs' supervisors received an information letter and a consent form for the Centre's participation (see Appendix A). If consent was given, the ECEs were then invited to participate. The ECEs who participated in this study received an information letter and the educators' consent form (see Appendix B). ECEs who were invited to

participate collaborated with no hesitation. I tremendously valued the ECEs' support, feedback, and suggestions while I visited the centres, and while I conducted the sessions.

The data was collected in two different research sessions using a variety of tasks: (1) a whole class session, which included two tasks (Task 1 and Task 2); and (2) an individual session, which included three tasks: a Symbolic Response Task (SRT), a Printing Reading Task (PRT), and a Reading Response Task (RRT).

Whole-class sessions involved all children from each age group. One class of 2-year olds, one class of 3-year olds, one class of 4-year olds, and one class of 5-year olds from each ELC submitted signed informed consent forms and agreed to participate in the whole class sessions. Subsequent to the whole class sessions, individual sessions were held with 16 children (two boys and two girls in each of the four age groups). These children were identified by the ECEs as representative of that age group.

Every child's parent(s) from the two ELCs received information letters and consent forms for whole class research sessions (Appendix C) and a total of 47 children agreed to participate. However, because of different circumstances (such as sickness and/or vacation time) only a total of 38 children were present during the whole class sessions. Parents of the 16 children invited to participate in the individual sessions received a second information letter and a consent form for the participation in the individual research session (Appendix D). Only those who gave consent participated in the sessions. It is my aim to share this study's findings with the ECEs and with the parents of the children from both ELCs who generously agreed to participate.

The Researcher's Role

As an ECE, I always have had a particular interest in how young children develop mathematical ideas and knowledge. In my more than twenty years of teaching experience with different age groups, I observed that mathematical domains develop gradually and, in most cases, children need time to accomplish and master different mathematical skills. This gradual process seems a key factor to be considered when teaching foundational mathematical skills to young children.

I believe that the first years of life are tremendously important in building core mathematical knowledge. My interest in young children's mathematical development led me to participate in different courses and conferences related to this topic. I was a member of a mathematical group guided and supported by mathematical researchers of the University of Rio Cuarto (Argentina). Adaptations and changes to a mathematical curriculum for three, four and five year-olds were the main topics of discussion and learning. Later, I became a member of the kindergarten writing curriculum committee in Prince Edward Island. During that opportunity, mathematics was the curriculum area I focused on the most. Throughout my Master of Education courses at the University of Prince Edward Island, early childhood mathematical education was the area I mainly explored in many of my research assignments. My teaching experience and my curiosity about how cognitive mathematical research could support the development of better early childhood mathematical practices have been the starting point for this qualitative study.

I ensured that all participants were treated ethically throughout this study. Because of the ages of my participants, I worked on building a trusting relationship with them both prior to and during each session. Play was a meaningful tool used to facilitate trusting relationships. Besides joining the children during play time, I participated in

other moments of the classes' routines (for example, snack time, outdoor time and circle time) and engaged in the children's conversations. I wanted to be sure that these young children felt confident enough to work with me in a particular classroom setting. As an ECE, I believe that caring and respecting young children are key elements for building a learning environment where they can feel safe.

The supervisors and the ECEs who agreed to participate in this study were informed daily regarding how the sessions unfolded. I reported on how the children responded, how the space settings worked, and how the schedules worked.

Sources of Data

I used numerous data collection tools for my study, including videotaping, observational records, children's notations, and my own reflective journal. The trustworthiness of the study was enhanced by these multiple sources that facilitated the rich description of the children's developmental trends and strategies while using symbols, printing, and reading numbers. It is my aim that the various sources of data and the thick descriptions of the context and the participants' actions and strategies (Lincoln & Guba, 2000) will provide a strong base that could facilitate the reader's decision about whether the findings of this study would apply to another context.

The exploration and interpretation of cognitive trends is supported by previous studies and previous cognitive frameworks that are described in the literature review (see Byalistok & Codd, 1996, 2000; Kato, Kamii, Ozaki, & Nagahiro 2000; Piaget & Inhelder 1971, 1983). The in-depth observations of children's strategies while using symbols, printing, and reading numbers is also supported by different studies and theories as well as my own experience and understanding of early childhood development.

Videotaping

My primary method of data collection was the videotaping of the tasks in both types of research sessions: the whole class session and the individual session tasks. The videotaping in this study was organized around the CGI framework. The reason behind videotaping the sessions was to facilitate the in-depth observation of children's sequence of responses. I was, therefore, able to develop a rich description of children's responses and strategies. Through the video observations I was challenged to use my own research knowledge (Frankee & Kazemi, 2001) in order to understand children's symbolic responses and strategies while representing, printing and reading numbers.

After the data was collected in both ELCs, I started to analyze the video. The possibility of observing the video multiple times facilitated the analysis of the types of children's responses and strategies. It is important to mention that while observing children's body language, children's emotions became noticeable (for example facial expressions). I therefore decided to also describe some of the emotions shown by the children while confronted with the tasks. Expressions such as confidence, frustration, and excitement are mentioned during the data analysis process.

Since the sessions were videotaped, it was not possible to guarantee the children's anonymity. Children's physical appearance is necessary to facilitate the in-depth exploration of body language strategies used by children while representing numbers (i.e., using finger counting, tapping, or looking for visual clues). However, to mitigate the risks, pseudonyms are used in the reporting of the findings.

In the future, after the study is completed, the raw video will be edited for pedagogical purposes. I expect to create a short video showing how young children

ranging in age from two to five represent numeric notations. The teaching video will be available for instructional purposes for ECEs. Permission to edit and use the video for pedagogical purposes was included in the informed consent form.

Observational Records

After reviewing the video, children's responses and strategies from the whole class sessions' tasks and from the individual sessions' tasks, were transcribed into the observational records I developed which recorded children's age, gender, and the amount represented. The observational records were organized as follows:

Figure 1Whole class research session

Whole class research session- Task 1 and Task 2
Age group:
Type of response Strategies

Figure 2 *Individual session- Symbolic Response Task (SRT)*

Individual Session
Symbolic Response Task (SRT)
Child's age: Gender:

Type of response Strategies

Figure 3 *Individual session- Printing response Task (PRT)*

Individual Session
Printing Response Task (PRT)
Child's age: Gender:

Type of response Strategies

Figure 4 *Individual session- Reading Response Task (RRT)*

Individual Session
Reading Response Task (PRT)

Child's age: Gender:
Type of response Strategies

RR#3
RR#9
RR#14

I wrote information from the whole class sessions based on group responses and strategies when the children were asked about numbers and numerals. In the observational records for individual sessions, I recorded information about children's types of responses and strategies from each of the individual tasks: the Symbolic

Response Task (SRT), the Printing Response Task (PRT), and the Reading Response Task (RRT).

Children's Notations

Children's numeric notations were collected during the PRT. Each child was required to print a numeric notation for three different amounts "3," "9," and "14.". The children's printed notations were organized as PRT#3, PRT#9, and PRT#14.

These printed notations are a rich source of information about how children from ages two to five represent numerals. The representations show how children construct the meaning of printed numbers. The representations are a visual record of how much children can accomplish when presented with paper and pen. The notations are a essential source of data in my study. Children's notations enrich my observational records because as Kato, Kamii, Ozaki, and Nagahiro (2000) state, they "externalize their thinking" (p.44). Examples of children's notations are included in the PRT section of Chapter 4.

Researcher's Journal

Throughout this study, I wrote about my own learning process in a journal. I noted feelings, thoughts, ideas, and graphic mind maps as well as different observations I did about the children. Writing became a crucial "method of inquiry" (Richardson, 2000, p.923) for me. Through my writing I constructed relationships not only about the children's responses, but particularly about my own knowledge construction process. The journal facilitated my continuous reflective process as well as the discovery of

relationships and trends that developed into new questions. Through my writing I engaged in the continuous process of making meaning of my study.

I scheduled weekly time during the writing process of this study to review my notes and to reflect on my own observations. Many of these reflections had an impact on the different decisions I had to make as a researcher during the data analysis component of this study.

Data Management

The different sources of data I collected for my study (videotaping, observational records, children's notations, and my own reflective journal) produced a large amount of information that needed to be organized carefully and with special consideration of details. In particular, I had to pay attention to the children's responses to the SRT, the PRT and the RRT.

The observational records were used to maintain the information collected during the whole class session and during the individual session. These observational records were completed after the multiple observations of the raw video.

The observational records for the whole class sessions (Task 1 and Task 2) considered the type of responses and the strategies used by the each class (see Figure 1). After the observational records for the whole class session were written, they were transcribed into a first draft summary table. This table recorded the responses of each age group (one table for the 2-year-olds, one table for the 3-year olds, one table for the 4-year-olds, and one table for the 5-year-olds). All the drafts for each of the summary tables were

condensed in a final summary table that included the contents of children's responses and strategies for Task 1 and for Task 2 (see Tables 1 and 2, Chapter Four).

Observational records were also developed for each task (the SRT, the PRT and the RRT) of the individual session (see Figures 2, 3, and 4).

The observational record for the Symbolic Response Task (SRT), considered the child's gender and age as well as the type of response and the strategies the child used (see Figure 2). For the Printing Response Task (PRT) three different observational records were designed (one for each of the amounts that children needed to represent: "3," "9," and "14"). These observational records considered the child's age and gender as well as the type of response and the strategies the child used (see Figure 3). Each of the child's printing responses was stapled to the appropriate observational record. This strategy ensured that records were well organized and easily accessible. The third observational record was used to record the children's answers for the Reading Response Task (RRT). The RRT observational record considered the child's age and gender as well as the type of responses, and the strategies the child used for each reading amount ("3," "9,"and "14") (see Figure 4).

All of the observational records for the individual tasks were organized by child. For example, Maria's SRT record, PRT#3 record (and the printing notation), PRT#9 record (and the printing notation), PRT#14 record (and the printing notation), and RRT record were stapled together.

After the observational records for each of the tasks of the individual session were written, they were transcribed into a first draft summary table. This table recorded the responses of each child. All the drafts for each of the summary tables were condensed in

a final summary table for the SRT that include the contents of children's responses and strategies by age (see Table 3), a final summary table for the PRT that include the contents of children's responses and strategies by age and amounts represented (see Table 5), and a final summary table for the RRT that include the contents of children's responses and strategies by age (see Table 7). The results recorded in each of these tables are discussed in Chapter 4.

The data was kept in a safe place and only my thesis supervisors and I had access to it. Children's names remained anonymous during the data management process. Transcribing these large amounts of data was time consuming. However, as a researcher I understood that rigorous organization of research transcriptions facilitated in-depth descriptions of the children's responses, and added to the accuracy of my findings.

Data Analysis

Throughout the study, the research questions guided my exploration and understanding of the children's responses. The main goal was to explore and report with clarity the developmental trends and the strategies young children used to name, represent, print, and read numbers. The findings from whole class and individual sessions were key components of the developmental framework used in this study in relation to how children understand representation, printing, and reading of numbers. The in-depth analysis considered both the children's types of responses when using symbols, printing, and reading numbers, and the strategies used by children when asked questions regarding numbers

After reviewing the video, I transcribed the observations and categorized them into different observational records. The possibility of reviewing the video multiple times facilitated my observation process, as well as the descriptions of the children's responses. The summary tables helped to organize the data, but also displayed the findings visually. Linkages and trends started to emerge by comparing the data in terms of children's ages, gender, and amounts.

Through Chapter Three I described the design and the theoretical framework adopted to conduct this study. To facilitate the reader's understanding of the methodology that was used, a description of the different sources of data was provided as well as details of how the data from the whole class session and from the individual session was collected, organized, and analyzed. In the next chapter, the types of responses and the strategies children used in all of the different tasks are described. Findings for each task are also summarized.

CHAPTER FOUR: RESEARCH FINDINGS

This chapter offers a rich description of children's responses and strategies used in the whole class session tasks as well as in the individual sessions tasks, the Symbolic Response Task (SRT), the Printing Response Task (PRT) , and the Reading Response Task (RRT). In addition, a discussion of findings is shared at the end of each task. A variety of children's numeric notations have been included in this chapter to enrich the descriptions and to facilitate the reader's understanding of the children's stages of development when printing numbers.

Introduction

Prior to starting the data collection process, I visited each ELC for one week. This facilitated my engagement with the children and allowed them to become familiar with me. The data collection process took place during one week in each ELC.

Two data-gathering sessions were planned: a whole class session, which included two tasks, and an individual session, which included three tasks. The purpose of the whole class session tasks was to explore the ideas and understanding that children ages two-to-five have about numbers and numerals; for the individual session the purpose was to describe the numeric notations and the strategies children ages two-to-five years old use, as well as their reading of those notations.

The whole class tasks and the individual tasks addressed the following research questions: (1) What kind of symbolic response do young children consider when prompted to represent numbers? (2) What strategies do young children use when prompted to represent numbers? (3) What kind of numeric notations do young children

use when asked to print numbers? (4) What are the developmental trends of young children's numeric notations? (5) What strategies do children use while printing numeric notations? and (6) What meaning do these notations have for the children?

In recording the findings, I organized the tasks by the children's ages to illustrate the progressions of their responses. The findings have been arranged in terms of types of responses and strategies used by the children. A brief discussion is summarized at the end of each age task. To facilitate the reader's understanding, the findings have also been recorded in one summary table for each task. I have included samples of children's printing to enrich my descriptions and clarify my observations.

Whole class session

Purpose

The purpose of the whole class session was to observe children's (1) types of responses and (2) strategies, such as meaningful actions or different body language approaches, when asked about numbers and numerals.

Protocol

In each ELC a whole class session was developed with all the 2-year-old children, the 3-year-old children, all the 4-year-old children, and all the 5-year-old children separately. A total of thirty eight children participated in the two tasks of whole class session. All of the children's parents received information letters and consent forms for the whole class research sessions. The responses were returned in most of the cases

within the time frame I had previously arranged with the ECEs. Only one child was not allowed to participate in the session. This child followed his regular class routines.

The session took 15 minutes in each age class and consisted of two tasks. I introduced the tasks during circle time and engaged in the normal routine that children follow during this time of the day. I explained the presence of the camera and the presence of the recorder to the children.

To facilitate the reader's understanding, I will describe the tasks, followed by findings in relation to (a) types of responses, and (b) strategies used by each age group. Children's responses for Task 1 and Task 2 were recorded in summary Table 1 and Table 2 respectively.

Whole class session. Task 1

I invited the children to count. I asked the children: "Who can count up to 10?" If children responded positively I asked: "Can you show me?" After they responded, I asked the group: "Can you count clapping your hands/ stamping your feet/snapping?"; "Is there any other way you can count?"

Table 1Summary table for the counting responses

Age	2	3	4	5
Type of response	None of the interviewed children responded numerically	Inconsistent responses. Some children started with the oral sequence "1, 2, 3," others didn't respond.	Most of the children in this group were able to count orally to ten.	All children counted orally to ten.
Strategies	Stamping and clapping was an action itself with no relation to counting.	Showing their fingers, counting orally to three or verbalizing indistinctly any number, word or picture. For those children who were able to count it was difficult to hear the oral sequence after "1, 2, 3." Also, the action of clapping or stamping was completed at a different time rhythm from the action of oral counting.	Children named the numbers clearly and loudly at the beginning of the sequence, "1, 2, 3," and at the end of the sequence "8, 9, 10." Many alternative ways of counting. Actions were completed at a different rhythm of rote counting.	Accomplished this task with no hesitation. Rote counting was sometimes accomplished by this age group as if they were singing. The children followed a rhythm as a group. They followed these rhythms with their bodies and heads movements while looking at each other and smiling.

2-year-olds Task 1 responses.

Number of participants: seven

(a) Type of response: none of the interviewed children responded numerically.

(b) Strategies: When asked to count in different ways (for example by clapping hands) the children remained silent. After a few seconds I asked again. One of the girls started to stamp her feet. Stamping was completed as an independent action, with no relation to the question. Overall, I was able to observe that these children did not seem to be interested in the activity.

It is important to explain that one of the educators came to the session with the group and coached her students' answers by giving prompts such as "Use your fingers,""Let's

jump" or "Can you count louder?" Only one of the girls followed the educator prompts and showed her fingers. The rest of the group remained silent.

3-year-olds Task 1 responses.

Number of participants: nine

(a) *Type of responses*: the children's responses were inconsistent; while some children started with the oral sequence "1, 2, 3," others did not respond at all.

(b) *Strategies*: Children answered using different types of strategies such as showing their fingers, counting orally up to three, or naming indistinctly any number, word, or picture.

When children clapped, the rhythm was followed differently by each child and the oral sequence was difficult to hear after "1, 2, 3." In the children who were able to count "1, 2, 3," it was observed that the action of clapping or stamping was completed at a different time rhythm from the action of oral counting. Overall, it appears that the children's attention was dispersed.

4- year- olds Task 1 responses.

Number of participants: ten

(a) *Type of response*: Most of the children in this group were able to count orally up to ten. "I can count up to 100," one of the boys said.

(b) *Strategies*: While counting orally, these children looked at each other. The speed of counting seemed to increase after one child's suggestion. The other children quickly followed him and started also to count faster. It was interesting to observe that

the children were able to name the numbers clearly and loudly at the beginning of the sequence, "1, 2, 3," and at the end of the sequence "8, 9, 10." When asked about other ways of counting, these children offered many different alternatives, such as, stamping their feet, covering their noses while repeating the sequence or suggesting the use of a hula hoop. These actions were completed at a different rhythm of rote counting. Overall, these children showed an interest in the activity as well as enjoyment while doing it.

5-year- olds Task 1 responses.

Number of participants: twelve

(a) *Type of response*: All children counted orally up to ten.

(b) *Strategies*: Oral counting was completed loudly and with a sense of "this is so easy." When asked about other ways of counting, these children offered many different alternatives and were able to accomplish this task as a group with no hesitation. For example, some children suggested kneeling down while counting or to combine many different actions and counting at the same time. After, reviewing the video for this particular age group, I perceived a sense of pride in these children while completing the task. They smiled while verbalizing their abilities for counting. "I can count up to 18", one boy said. "I can count to 100" replied one girl. "Counting 5 is easy…I am five," suggested another boy.

Rote counting was sometimes accomplished by this age group as if they were singing. The children followed a rhythm as a group, usually started by a leader. They followed these rhythms with their bodies and head movements while looking at each other and smiling.

Whole class session. Task 2

I invited the children to look around their classroom/ room/area and asked them: "Who can look around and show me a number?"; "What number is it?"

Children's responses were recorded in summary Table 2.

Table 2 Summary table for the numerals responses

Age	2	3	4	5
Type of response	Different written notations were pointed out as numerals	Different written notations were pointed out as numerals	The responses were inconsistent. Some children were able to identify numerals with accuracy; others pointed at any written notation	All children were able to name accurately most of the numerals they pointed to
Strategies	Some children looked up different written notations and pointed at them.	Some children stood up from the circle and looked for labels on the walls.	Some children stood up from the circle and looked for labels on the walls	All children stood up from the circle and looked for numerals and written symbols placed on the walls, books, and in the keyboard. Peer exchanges of knowledge were a common response

2- year-olds Task 2 responses.

Number of participants: seven

(a) *Type of response*: different written notations were pointed out as numerals; for example one child pointed at number "46" while another child responded orally "two fishes" while pointing at a buoy that was hanging on the wall (the numeral "46" was printed in the buoy).

(b) *Strategies*: When asked about numerals, this group of children looked up the

walls in search for different written notations and pointed at them. Some of their responses were:

"This" (pointing at the number 46 was printed in a buoy)

"This, and this, and this" (pointing at different labels founded on the room)

"Two fishes" (pointing to another label)

"This" (pointing at a label with a teacher's name)

3 –year-olds Task 2 responses.

Number of participants: nine

(a) *Type of response*: different written notations were pointed out as numerals.

(b) *Strategies:* When asked about numerals, some children stood up from the circle and looked for labels on the walls. Some of the responses included:

"This one" (pointing at the number 1)

"That one" (pointing to any label on the wall)

Generally, I was able to observe that these children's interest in the task was brief and that their attention was dispersed.

4- year-olds Task 2 responses.

Number of participants: ten

(a) *Type of response:* the responses were inconsistent. While some children were able to identify numerals with accuracy, others pointed at any written notation.

(b) *Strategies*: When asked about numerals, some children stood up from the circle and looked for labels on the walls. Some of the responses included:

"All of them" (pointing on a keyboard to the numerals 1 to 0). "One, two, three, four, five, six, seven, eight, nine, ten, zero."

"Eight; nine." (pointing at the keyboard)

One child pointed at the letter "O." Immediately, one of the girls said, "That is an "O", "not a zero." Another child pointed to the letter "C", to which a peer responded: "That's not a number; it is a letter. While searching through the classroom, some children pointed at the EXIT sign. Other children pointed to the big clock that was on one of the walls. A small group started to search for their first names in an area that has individual assigned lockers.

5- year -olds task 2 responses.

Number of participants: twelve

(a) *Type of response*: All children were able to name accurately most of the numerals they pointed to, such as "two," "nine," "three," and "five." It was interesting to observe how most children looked for the numeral "five", which appears to be very popular (perhaps because it responds to this group's age).

(b) *Strategies:* When asked about numerals, all of the children stood up from the circle and looked for numerals and written symbols placed on the walls, books, and keyboard. Sharing ideas and "correcting" others' responses was observed continuously, particularly from the girls to the boys. The following situation describes these interesting peer exchanges of knowledge:

"One," said one of the boys pointing at the keyboard.

"That's not a "one," it's an "I."

A few seconds later, one boy said: "Six," pointing at the number stamped in a buoy. Some children looked for numbers that were printed in small font and that were located in places not very easy to see. For example, the number of players written in a board game box, the time located on the left side of the computer screen, or the small numbers printed on an open book.

Discussion for Task 1 and Task 2 responses

As demonstrated in the literature review, young children have some familiarity with numbers and numerals as clearly shown by the whole class tasks. It seems that this familiarity increases with age, particularly between the ages 4 and 5.

After being asked to count, children ages 3 to 5 spontaneously rote counted, following a rhythm that engaged the whole group. For the 5-year- olds the counting was almost like singing. For this age group, the sequence appears as something that children know and that is easy to repeat.

Naming the words for each number became more accurate as the children's ages increased. For example, the 3-year-olds showed that "1, 2, 3" was what they knew the best; they said these three words loudly and with confidence and, as the sequence continued, their voices decreased. The 4–year-olds said the beginning of the sequence ("1," "2," "3") loudly and confidently, as well as the end of the sequence ("9," "10"). The 5-year-olds were able to name accurately and loudly all the words in the sequence.

It appears that the rote counting is learned as a sequence of memorized words (one number word after another). One interesting observation refers to the memorized oral routines of counting that children ages 4 and 5 seemed to have. These children

followed the rhythm of the sequence with their body movements (i.e. moving their heads) even if they didn't know some of the number words. For example, when children didn't remember the words "six," "seven," "eight," they mumbled three sounds and then continued until they said loudly "nine," "ten."

The 3-year-olds paid special attention to the beginning of the sequence ("1, 2, 3"). A possible explanation is that this age group relates to what they know the best. Perhaps the three words, "one," "two," and "three" are part of the perceptual understanding of numbers described by Piaget (1952) but also, the first step to memorizing the oral routine of counting.

The last number in the sequence was named loudly by 4 and 5-year-olds. According to Siegler (2003), this tendency demonstrates that these children have started to consider the value of the last word as referring to a set, in this case a set of ten. According to this researcher, this is an important step towards the understanding of cardinality.

Overall, the children I interviewed were familiar with the routine of counting with the exception of the 2-year-olds. Bialystok and Codd (1996) and Hughes (1986, 1991) suggested that the familiarity of counting that young children have does not mean that they understand the meaning of the numbers. However, while most of these previous studies seemed to consider this familiarity as a not meaningful response, I believe that the understanding of number words and sequence is knowledge that children do have which is extremely valuable from a pedagogical perspective.

Important consideration needs to be given to the routine of rote counting. Rote counting appears to be a universal skill children accomplish (Siegler, 2003). Based on my

teaching experience, ECME tends to rely heavily on the skill of counting orally and, therefore, counting activities are a main component of mathematical practices. However, mathematics is not only about oral counting and recognizing numerals. The pedagogical implications for the rote counting routine will be discussed in Chapter Five.

As the children's ages increased, there seemed to be not only an increase in the naming but also in the recognition of numerals (the conventional written form of numbers).

Most children who were interviewed (including the 2-year-olds), seemed to know that they have to look for printed material when asked to find a number in the classroom. It did not matter if the pointed symbol was a written sign (i.e., "EXIT"), a letter, a name, or a numeral, all the children's responses refer to written symbols. Therefore, it could be assumed that these young children understand that numbers can be written. The difference in the responses depend on how children identify numbers from pictures and numbers from letters.

In conclusion, when asked to count and to find numerals, children seemed to be familiar with the numerical discourse and with questions such as "Can you count?" and "Can you look around and show me a number?"

Individual session

Individual sessions were held with 16 children, a boy and a girl from each age group at each of the two ELCs. Children who exhibited typical behaviours for each age group were identified by ECEs. Parental informed consent for each child's participation in the individual session was obtained separately from the whole class

session. The responses were returned, in most of the cases, within the time frame I previously arranged with the ECEs. Only one child was not allowed to participate in this session; in this case, the ECE was asked to choose another child using the same criteria.

The sessions took place in a quiet room previously chosen by the ELC's supervisor. The individual session consisted of three different tasks as outlined below:

1. Symbolic Response Task (SRT) to observe what kind of representation the children used other than oral words.
2. Printing Response Task (PRT) to record the types of numeric notations and strategies children used while printing three different amounts (3, 9, and 14). The responses following this task are described separately as PRT#3, PRT#9, and PRT#14. A sampling of children's printing work has been included in the PRT section.
3. Reading Response Task (RRT) to observe children's types of responses and strategies used when reading their own notations. The responses following these tasks are described separately as RRT#3, RRT#9, and RRT#14.

Symbolic Response Task (SRT)

Purpose

The purpose of the SRT is to observe what kind of representation children choose and what strategies children use when prompted to represent the number of a meaningful situation (their own age).

Protocol

After welcoming the child to the room and discussing the presence of the camera, I invited him/her to play a game where no words were allowed. I asked the child "Can you show me how old you are?" With this question, the child was asked to use a way other than oral communication to express his/her age. For example, a child might clap four times or line up four blocks to represent four years of age. Empty cards, markers, and manipulatives were at the table in case the child chose to print or use the manipulatives to represent the amount.

Each child was invited to play a game where they had to show me, without using words, how old they were. Because of the children's ages, I asked them a few times and if they were not successful, I asked them to tell me how old they were. Based on my teaching experience, almost all young children are able to verbalize their age, therefore concluding this section with a doable task that allowed those children to feel successful.

To facilitate the reader's understanding, I will describe the tasks and follow that description with the findings in relation to (a) types of responses and (b) strategies used by each age group.

Children's responses were recorded in summary Table 3.

Table 3 Summary table for the Symbolic Response Task (SRT)

Age	2	3	4	5
Type of response	Two children responded accurately by using fingers and words. Fingers were used all at once Two children did not respond.	One child responded accurately with words. One child responded accurately using fingers and words. One child responded accurately using fingers. Fingers were used all at once One child affirmed with his head that he was 3 after being asked.	Three children responded accurately using fingers. Fingers were used all at once/ One child responded orally after being prompted twice.	All children responded accurately using fingers. Fingers were used all at once/
Strategies	Fingers and words used simultaneously	Fingers and words used simultaneously	Two participants tapped or moved their four fingers at the table while I asked the question. Three children responded quickly and with no hesitation.	All children looked at the adult while the question was formulated.

2- year- olds SRT responses.

Children interviewed: Julia, Gloria, Connor, and Alex

(a) *Type of representation*: After being prompted twice, Julia and Connor could not show me their age. "I don't know" was the verbal answer I received from Julia, while Connor looked away. The other two participants, Gloria and Alex, used their fingers with accuracy and at the same time responded loudly: "Two."

(b) *Strategies*: Gloria and Alex used fingers and words simultaneously. Fingers were shown at once.

It appears that the girls seemed more relaxed and expressed certain interest in the "game," while the boys looked more serious and/or shy. This was observable in their body language, as the girls smiled and seemed to have fun, while the boys appeared tense and showed little enthusiasm.

3- year-olds SRT responses.

Children Interviewed: Maria, Laura, Ben, and Max

(a) *Type of representation*: In the case of the girls, Maria responded orally "three", while Laura used her fingers with accuracy and at the same time responded orally "three".

Ben didn't respond to the first prompt and when prompted again, he affirmed with his head that he was three. Max used his fingers accurately.

(b) *Strategies*: The oral responses were simultaneous with the action of showing the fingers. Fingers were shown all at once. All four participants appeared serious and worried until the question was formulated; after that, they answered and looked away. In the case of Ben, his eyes looked down when I waited for him to respond.

4- year-olds SRT responses.

Children interviewed: Cathy, Celia, John, and Chad.

(a) Type of representation: Cathy, Celia, and Chad used their four fingers; John responded orally "four" after I prompted him twice.

(b) Strategies: All children paused after the prompt as if they were

thinking. Fingers were shown at once. The children appeared serious until the question was formulated. In the case of the girls, Cathy and Celia tapped or moved their fingers at the table while I asked the question. During the video reviewing, I was able to observe that, except for John (whom I prompted twice), the rest of the children appeared relaxed and comfortable with the activity. After their response, the three children that responded to the first prompt smiled. These three children responded quickly and with no hesitation.

5- year-olds SRT responses

Children interviewed: Karle, Heather, Zack, and Carlos.

(a) *Type of representation*: all children used their five fingers to represent their age.

(b) *Strategies*: All children looked at the adult while the question was formulated. With no hesitation, these children showed all fingers at once.

The girls seem more relaxed than the boys and smiled with a confident attitude while I formulated the question. The boys looked more serious and tense with little enthusiasm to play the game.

SRT discussion

Knowing how old you are is an important concept for most young children and it is usually expressed by them with pride. This concept has been mostly taught and socially transmitted by adults to children from very young ages. It is difficult to determine how this concept is learned by children, but when young children are asked "how old are you"? The symbolic response (mainly oral language) and the quantity are linked correctly. Symbolic responses are considered by this study as those that involve the use of any type of symbol to represent a quantity; for example, words, printing, or the

use of manipulatives. By asking the children to show me, I challenged them to avoid the use of words and find another symbolic way to answer.

It was interesting to observe that even though markers, paper, and manipulatives were present, none of the interviewed children used these tools to represent symbolically; they preferred to rely, particularly at ages 4 and 5, on their fingers. Eight of the interviewed children chose their fingers to accomplish this representation and three children (ages 2 and 3) also combined this action with words. Fingers were, therefore, the most common type of symbolic response that the children who were interviewed considered. In all these cases, the responses were accurate; that is, children used two, three, four, or five fingers to represent the actual number for their ages.

When I asked the children to "show me," I was able to observe, especially from ages 3 to 5, how children's body language communicated their level of understanding to the challenge of not being able to use words for their responses. For example, knowing how to respond was expressed with a smile and by a relaxed attitude, while not knowing how to answer was expressed with serious faces and tense bodies.

Overall, a sense of "I know the answer" was expressed as the children's ages increased. Smiles on their faces, tapping the right amount of fingers while I formulated the question, and showing their fingers with confidence were some of the strategies observed in most four and five year olds.

In two and five year olds, there appeared to be a pattern of responses based on gender. Most of the girls who were interviewed seemed more enthusiastic than the boys about this activity. Moreover, a certain level of confidence was expressed by the girl's body language, while the boys appeared more tense and serious during the sessions.

Printing Response Task (PRT)

Purpose

The purpose of the PRT was to observe and describe in detail children's (a) types of numeric notations and (b) children's strategies while printing numbers.

Protocol

The printing task is partially adapted from Bialystok and Codd (1996) whose study explored the linkages between cardinality and written numbers. In this task, children were asked to write on a card the amount of objects they could see in a box. Each child was asked to examine three boxes containing 3, 9, and 14 objects respectively. The different amounts selected explored the notations children use in relation to:

- numerals that represent a small amount; ("3");
- numerals that represent a larger amount ("9"); and
- double digits numerals ("14").

There were multiple types of objects in each box. Various small items such as a plastic fish, a bat, a sheep, a cube and a flower were placed in each box. This approach was chosen to observe children's responses to the question "how many" regardless of the set's characteristics. In other words, I intended to explore children's responses when presented with a set where the objects' attributes and values were different from each other. Typically, sets are presented to young children with one common qualitative characteristics (all flowers, all red, all crayons), as if this was the only way to organize sets for counting and talking about numbers. In reality, however, environments are composed of different objects that could become a set.

Children's printing notations were observed and described in detail. By repeatedly reviewing the video, I was confronted with the issue that most of the participants' responses did not completely fit into categories of previous studies. For this reason I developed a table (with partial adaptations from previous studies: Hughes, 1998; Sinclair, Siegris, & Sinclair, 1983, as cited in Teubal & Dockrell 2005) that considers stages of the children's number printing. In this table, I included categories such as mathematical verbalizations ("bunch," "more," "less"), representations that resemble accuracy from those that do not, as well as a description for when the child represents a quantitative idea for the first time.

Table 4 Children's number printing stages

1-Idiosyncratic	Use of scribbles. The representations are ambiguous.
2- Objects representation	Qualitative representation of the set. The child represents the objects or some of the objects attributes / values
3- Global representation of quantity	The child represents, for the first time, a fragile quantitative idea. Accuracy is not considered. The child represents and also verbalizes the idea of: a "bunch," "more," "less", etc. The child uses (a) qualitative representations (drawing the objects) or (b) analogic marks (i.e., tallies, dots)
4- One-to-one representation of quantity	The child accurately represents the amount of a set by using (a) qualitative representations (i.e., drawing the objects) and/or (b) analogic marks (i.e., tallies, dots)
5- One-to- one representation of quantity using numerals	The child uses conventional numerals or number-like-forms one-to-one. For example "123" or "111" to represent "3"
6- Number-like form	The child use symbols that looks like numerals. The child considers the quantity and uses a written symbol that look like a numeral to represent it. For example, the child prints an "E" for "3" or prints a "not perfectly" traced "3"
7- Conventional printing	The child prints numerals. Quality of printing could be considered.. The numeral is not accurate with the amount. Sometimes a mirror form of the numeral is traced
8- Cardinal printing	The child prints a numeral that displays the cardinal meaning of the set. Quality of printing is important.

The pedagogical implications of this table will be discussed in Chapter Five.

During the task the child was allowed to explore and manipulate the items in each box before receiving an empty paper and a pencil. The child was asked then to write something on the card that could help them to remember how many items were in the box. The procedure was repeated with each of the three boxes. After that, the child went back to his/her class.

To facilitate the reader's understanding, I will describe the findings of each represented amount (3, 9, and 14) in relation to (a) type of representation, and (b) strategies used by each age group. I have also included some of the children's printing to enrich my descriptions. These representations were randomly chosen from each age group.

Children's responses by amount are summarized in Table 5 (the bold text refers to strategies).

Table 5 Summary table for the Printing Response Task (PRT)

Age/ Amount	2 year olds	3 year olds	4 year olds	5 year olds
3 objects	Three idiosyncratic representations One representation is not related to the quantity in question **Described, name objects** **Appeared to concentrate while tracing (with one exception)** **Rudimentary grasping**	Three idiosyncratic representations (one child used scribbles; two small marks) One -by-one representation (tally marks) **Objects get lots of attention** **Appeared to concentrate while tracing**	Two one- to- one representation of quantity (one tally marks, one of objects) One number-like form (one E) One conventional printing (in mirror form) **The girls seem to have the need to look at the objects;** **The boys have certain knowledge about 3;** **Grasping has improved**	One number -like form Three cardinal printings **Quality seems to be important** **Grasping is well developed**
9 objects	Three idiosyncratic representations One representation is not related to the question **Described, name objects** **Appeared to concentrate while tracing (with one exception)** **Rudimentary grasping**	One Idiosyncratic representation Three global representations (objects and tally marks) **Use of words like "lots"** **Checking back at the objects** **Grasping is fragile**	One idiosyncratic representation One global representation (tally marks One one-to- one representation (objects) One cardinal printing **Oral counting** **Checking back at the objects** **Grasping has improved**	Two cardinal printings representations One conventional representation One Number-like form **Oral counting** **Seemed to be thinking before printing** **Grasping is well developed**
14 objects	Three idiosyncratic representations One representation is not related to the question **Described name objects** **Appeared concentrated while tracing (with one exception).** **Take objects outside of the box** **Rudimentary grasping**	One idiosyncratic representation Three global representations (objects and tally marks) **Oral counting** **"There is lots here", "there is going to be lots to do"** **Grasping is fragile (one exception)**	One idiosyncratic representation One global representation (tally marks One global representation (objects, not completed) One number- like form **Oral counting** **Checking back at the objects** **Touching with two hands** **Sense of frustration** **Grasping has improved**	One Cardinal printing representation Two conventional printing representations (no accuracy One no response **Oral counting (whispering)** **Count orally while touching objects** **Counting orally while looking at the objects**

PRT#3

2- year-olds PRT#3.

Children interviewed: Julia, Gloria, Connor, and Alex

(a) *Type of response*: Julia, Connor, and Alex used idiosyncratic types of responses. Gloria looked at the box and said "there are three." Then she printed something she knew: the first letter-like form of her name."G" she said. "G for Gloria."

(b) *Strategies:* in relation to the objects, Julia, Gloria, and Connor explored the items and verbalized some of their characteristics such as "is red," "a little fish", or asked "what is it" after looking at each object. In terms of the printing, these children seemed relaxed and appeared concentrated while tracing. Alex appeared tense and dropped the marker after finishing. All of the interviewed children used the whole hand forming a fist around the marker.

Figure 5 Julia's idiosyncratic representation of "3"

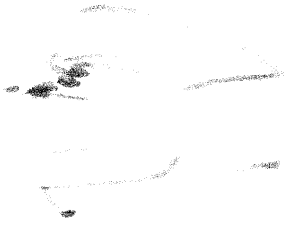
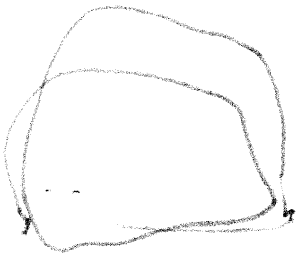


Figure 6 Connor's idiosyncratic representation of "3"



3- year-olds PRT#3

Children interviewed: Maria, Laura, Ben, and Max

(a) *Type of response:* Maria, Laura, and Ben used idiosyncratic representations; Maria and Laura used small marks while Ben used scribbles. Max did a one-to-one representation through tally marks. This last response was accurate (three lines for three objects).

(b) *Strategies*: In terms of the objects, the girls removed the items from the box; Ben asked adult permission to do the same. Max kept the items in the box and checked continuously by looking at the items while printing the tally marks. He remained silent while completing this action. Maria used several words to either describe, name or ask about the objects, for example:

"This is pretty."

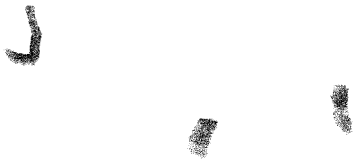
"What is this?" she asked while looking at a bat. "It's a bat! she exclaimed a few seconds later.

"I am going to do the little fish now..."

Regarding the printing strategies, with the exception of Ben who used scribbles, the children printed small marks, showing a preference for straight lines. All of the children appeared to be concentrating while printing and happy with their responses.

Figure 7 Maria's idiosyncratic representation of "3"

Figure 8 Max's one-to-one representation of "3"



4- year-olds PRT#3

Children interviewed: Cathy, Celia, John, and Chad.

(a) *Type of representation*: The girls used representations that involved the consideration of representing one-by-one. Cathy represented "3" by using three tally marks. Celia represented "3" by drawing each object that was in the box with plenty of detail. John and Chad were the first children interviewed who considered the possibility of representing an amount by using a symbol that looked like the conventional written form for "3." John printed a "3" in mirror motion (going from right to left) while Chad printed a form that looked like letter "E."

(b) *Strategies*: Each of the children used different strategies when representing "3." The girls seemed to have the need to look at the objects; Cathy took the objects out of the box, and Celia looked at specific details while drawing. Celia kept the objects in the box and took the time to observe them before she started to print. Cathy asked if she could use lines to represent "3." She seemed relieved when she was allowed to do this.

The boys, on the other hand, seemed more comfortable with the knowledge they have about "3"; John nodded positively with his head while printing and smiled proudly when he finished the task. Chad proudly counted orally "1, 2, 3," while touching each of the objects. When he printed a form that looks like the letter "E" he made a noise for each of the horizontal lines this letter has. None of the boys took the objects out of the box.

Grasping has improved tremendously from the previous age group. The children's facial expressions showed an understanding of the questions and the task. In some cases, they assumed a "thinking" position (as if they were concentrating on the task) before starting to print.

Figure 9 Celia's one-to-one representation of "3"

Figure 10 John's conventional numeral representation of "3"

Figure 11 Chad's number-like form representation of 3

5 year-olds -PRT#3

Children interviewed: Karle, Heather, Zack, and Carlos.

(a)Type of response: Karle printed a number-like-form for "3" while Heather, Zack, and Carlos used a cardinal representation to represent the three objects in the box. *Strategies*: In the cases of Heather and Carlos, both children said the number orally before printing the digit. Karle looked at the objects, counted, and then started to print. She stopped at the first half circle while printing "3" (seemed to be thinking), and then continued with the other half circle. Her final product looked like a "3." Heather, Carlos, and Karle smiled while I explained the task. They appeared comfortable with their responses. Zack, on the other hand, seemed nervous. After looking at the box with three items he said: "I don't know how to make the right number." Then he started to trace on the paper, looking at the adult for approval. He seemed unsure and worried.
Overall, grasping was well developed in all children.

Figure 12 Karle's number -like form for representation of "3"

Figure 13 Heather's cardinal representation of "3"

Figure 14 Zack's cardinal representation of "3"

PRT#3 discussion

In response to the task "write on the paper," the 2-year-olds seemed to understand the task in the sense of printing or marking. This age group showed no anxiety or worries about the types of answer they gave, either in terms of quality or in terms of accuracy (even in the case of one child who was able to verbalize "there are three"). It appeared that intentionality, as a main characteristic of individual's symbolic representation, is not the purpose of these children when asked to write about quantities. Gloria knew that "G" was the first letter of her name; therefore, her response differed from the rest of the children in the sense that she had an intention to represent this letter. However, in terms of the task, there seemed to be not consideration about representing the idea of how many.

The 2- year-olds showed a need to describe and name the objects. "What is it?" asked Julia every time she took an object out of the box. While printing, most of the children enthusiastically concentrated on the activity while their eyes followed the motion of the traces.

It was interesting to observe that for the first time, the amount seemed to affect the representations that the 3-year-olds made. Representing "3" appeared to be supported by the major role some of these children gave to the objects in the set. (i.e., tracing the box and tracing or drawing each item's characteristics. With the exception of Ben who used scribbles, the children printed small marks, preferring the use of straight lines.

The observations suggested that there is an understanding of the value of "3" in the 4-year-old group. Even though the girls' responses were different from those of the boys, the idea of three objects was easily represented either by tally marks and picture drawing

or by the intention of printing conventional or forms that look like "3" (in the case of the boys). As a small number "3" objects seem to be easy to touch, look at, and also easy to capture by looking at the set only once.

The ways these children chose to represent "3" differed in quality, accuracy and perhaps in the acquired knowledge, culturally transmitted by adults, about what "3" looks like. Overall, the numeral "3" appeared to be well known when I asked this age group to write how many objects were in the box.

All 5 -year-olds showed an understanding of the use of conventional numeric symbols to represent quantity. The participants seemed to know that by using one written symbol they could represent the total amount in the set, in this case "3." This age group seemed to give important consideration to the quality of the tracing of number 3. This consideration could be an important step towards the understanding that the numerals have meaning. Therefore, in order to be able to communicate that meaning, the printed symbol had to be well traced so it could be read by others.

PRT#9

2- year- olds PRT#9 responses.

Children interviewed: Julia, Gloria, Connor, and Alex

(a) Type of representation: Julia, Connor, Gloria and Alex used idiosyncratic types of responses. Gloria once again printed something she knew: the first letter of her name.

(b)Strategies: in relation to the objects, Julia, Gloria, and Connor took the objects out of the box and distributed them with no order on the table. Julia said, "Is empty" after taking the objects out of the box. The girls named each of the objects while taking them

out of the box. In terms of printing, Julia, Gloria and Connor seemed relaxed and also appeared to be concentrating while tracing. Alex appeared tense and dropped the marker after finishing. All of the children interviewed used rudimentary grasping while printing.

Figure 15 Gloria's idiosyncratic representation of "9"

Figure 16 Alex's idiosyncratic representation of "9"

3- year- olds PRT#9

Children interviewed: Maria, Laura, Ben, and Max

(a) *Type of representation*: Maria printed a global representation of the quantity. She intended to represent the different objects in the set; however, accuracy in terms of the quantity was not accomplished. Laura used an idiosyncratic representation. The boys' representations were also different from each other. While Max continued to use tally marks, this time he combined them with circles and drew them one after the other (one tally mark-one circle-one tally mark-one circle). These analogic marks were not accurate and the representation was global. Ben represented the objects from a qualitative perspective by tracing the box and drawing scribbles inside it.

(b) *Strategies*: Maria supported her need to name and describe the objects, by drawing each of them. She expressed terms that relate to quantities such as "there is lots on this one," "a lot," "there is a lot" a number of times. Laura, instead, preferred to line-up the objects while printing; she kept looking (checking back) at them while completing the task. At the end, she traced one big circle. Max also chose to look and examine the objects while printing; however he kept the items in the box. Ben seemed quite confident by discovering the opportunity to trace the box. In all cases, grasping was still quite fragile. All children in this age group seemed quite comfortable with their responses.

Figure 17 Maria's global representation of "9"

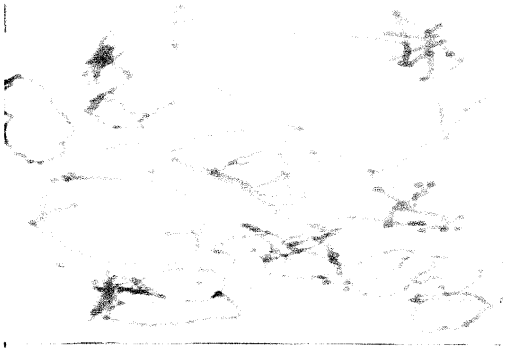


Figure 18 Laura's idiosyncratic representation of "9"

Figure 19 Ben's global representation of "9"

4 –year- olds PRT#9

Children interviewed: Cathy, Celia, John, and Chad.

(a) *Type of representation*: Cathy and Celia continued to use the same type of representation they used when representing three objects: tally marks and detailed drawing respectively. However, this time Celia's representation was not accurate while Cathy was (she drew the nine objects with many details). John's type of response changed with the larger amount. This time he used an idiosyncratic representation as compared with the previous representation of the number "3." Chad used a number-like-form for "9" that he printed very small.

(b) *Strategies*: Cathy, Celia, and Chad seemed quite happy with their responses, and the strategies they used to solve the task appeared consistent with the strategies they used in the PRT#3. Cathy used tally marks and eventually looked back at the objects in the box while Chad counted orally touching the objects one-by-one before printing. It was interesting to observe that because of her concentration on each of the object details, Celia took a few seconds to look inside the box and touch the objects before printing. She kept looking at them while drawing.

John's strategies changed completely in comparison with the previous task. After looking at the box with nine objects, he seemed alarmed and overwhelmed. He looked down and looked inside the box again. After tracing the mark (a closed form scribbler) he looked at the adult. He seemed to not be happy with his response and his face showed discomfort.

Figure 20 Cathy's one-to-one representation of "9"

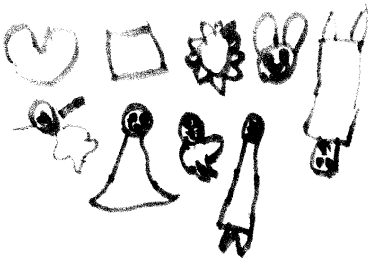


Figure 21 Chad's number-like form representation of "9"

5- year- olds PRT#9

Children interviewed: Karle, Heather, Zack and Carlos

(a) *Type of representation*: Two children in this age group used the cardinal numeral and one child used a number-like form to represent the nine items.

(b) Strategies: all of the children who were interviewed counted before printing. Karle, Carlos, and Zack counted while moving their eyes from one object to the next; Zack stood up while counting with his eyes; Heather instead, counted by touching each object. These children did not take the objects outside the box and, as demonstrated by their body language, they all seemed to be thinking before they eventually started to print. The children's facial expressions showed an understanding of the questions and the task. They all seemed proud of their responses.

Figure 22 Heather's cardinal representation of "9"

Figure 23 Karle's number-like form representation of" 9"

Figure 24 Carlo's conventional representation of "9"

RRT#9 discussion

The 2- year- olds interviewed maintained the same type of printing responses they used previously with the amount "3." Once again, in response to the task "write on the paper," the children seemed to understand the task in the sense of printing or marking. When representing "9," this age group showed no anxiety about the types of numeric answers either in terms of quality or in terms of accuracy. With the exception of Alex, the children enthusiastically concentrated on the activity.

At three years of age, it appeared that the children understand that the amount had changed; therefore, the idea of "lots" needed to be written on the paper. Accuracy seemed not to worry these children; however, the value of representing "lots" appeared as important and manifested through, for example, the representation of many tally marks. It could be theorized that when children explored and printed "9," they were comparing this amount with the one they had explored before ("3"); thus "9" appeared as "more" and as "lots. "

All of the 4-year-old participants used the strategy of continuously referring to the objects while printing. Overall, it seemed that these children did not lose track of the objects already represented in the case of representing "9." None of these children removed the objects from the box, and the exploration was completed before printing by touching, counting orally, or by putting, like Celia, almost all of her head inside the box.

It is interesting to observe that the 4- year-old girls used the same type of representation for "9" that they did when asked previously to represent "3." One interesting component of the two girls' representations, was that while Cathy used a more developed form of representation, her answers were not accurate (she printed seven tally

marks).Celia, on the other hand, used a qualitative representation for each object, and responded with accuracy when drawing nine objects. Perhaps the possibility of relying on the objects facilitated the consideration of the details thus allowing Celia to represent "nine" correctly while Cathy's tally marks, as analogic marks, were difficult for tracking when checking back at the objects.

I observed that, a shift in the responses happened in the 4-year-old boys. It appeared that both of them considered the socially transmitted knowledge of what numbers should look like. Knowing this, but not being able to print it correctly, seemed to cause a sense of discomfort in John, based on his body language expression. This example could suggest that, for the first time quality when printing numbers (Teubal & Dockrell, 2005) is considered in this study. Another interesting observation that arose from this age group, particularly from John's response, is that he did not use any kind of analogic marks (i.e., tallies, dots) but instead relied on an idiosyncratic representation (a closed trace form). This representation could show that John has an understanding of how a set is socially represented (by *one* symbol). In other words John, (and Chad as well) could already understand the inclusive notion that cardinality proposes. As explained in the literature review of the study, these boys could understand that "9" includes 1, 2, 3, 4, 5, 6, 7, 8, and 9, and that printing the last number involves the whole set.

In the case of the 5-year-olds, one written symbol was used to represent the total number of items in this set "9." Even though two of the participants used number-like-forms (prints that looked like the number "9"), it was clearly suggested that these children seemed to understand that the numerals have meaning and that they referred to the specific quantity of a set.

At this age, knowing what the numeral looks like but not being able to print it seemed to create a certain level of frustration that apparently is solved by different personalities in different ways.

All of the 5-year-old children counted both orally and by touching the objects before printing. These children did not take the objects outside of the box

PRT#14

2- year- olds PRT#14

Children interviewed: Julia, Gloria, Connor, and Alex

(a) *Type of representation*: Julia, Connor, and Alex used idiosyncratic types of responses. Gloria once again printed something she knew: the first letter of her name.

(b) *Strategies*: in relation to the objects, Julia, Gloria, and Connor took the objects out of the box and distributed them randomly on the table. Julia and Gloria named some of the objects while taking them out of the box. In terms of printing, Julia, Gloria and Connor seemed relaxed and also appeared to concentrate while tracing. Alex appeared tense and dropped the marker after finishing. All children used rudimentary grasping while printing.

Figure 25 Julia's idiosyncratic representation of "14"

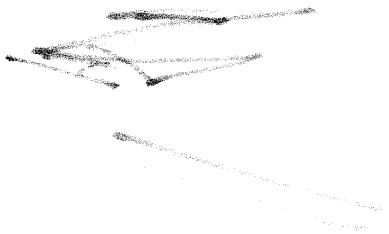
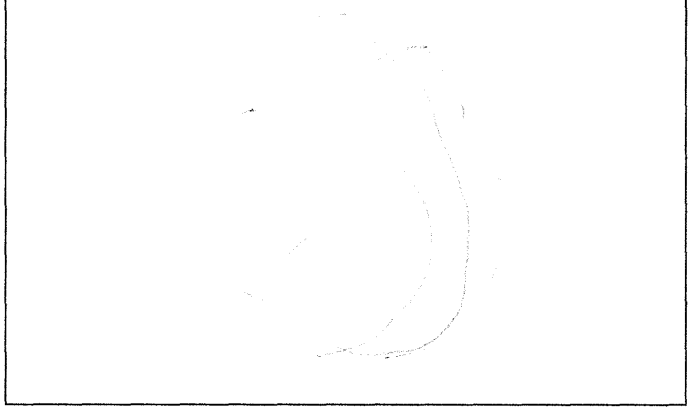


Figure 26 Alex's idiosyncratic representation of "14"



3 year olds PRT# 14

Children interviewed: Maria, Laura, Ben and Max

(a) *Type of representation*: Maria printed a global representation of the quantity; she represented the different objects in the set by tracing a circle around some of them (some objects were considered more than once). Laura used an idiosyncratic representation. The boys' representations followed the same patterns they used when representing nine items. Max traced tally marks that did not represent the amount accurately; however, this time he represented a few more tally marks than when he represented "9." Ben, once again, represented the objects from a qualitative perspective by tracing the box and drawing scribbles inside it. However, this time he counted orally up to nine and while counting, stamped dots inside the traced box. The pattern of

counting orally and stamping did not follow the same rhythm; many dots were represented inside and outside the traced box.

(b) *Strategies*: Again, Maria supported her need to name and describe the objects by drawing a circle around each of them. In some cases she drew a circle around the same object several times. However, after a few minutes, her face started to change and she seemed to be overwhelmed. Maria lost track of which objects she had already traced. She seemed confused and tired. She decided to stop. While printing, she expressed "there is lots here," "there is going to be lots to do." Laura used the same strategies she used before: she took the objects out of the box, lined them up and looked at them while tracing a circle. This time she spent more time completing the activity as she coloured inside the circle. The grasping seemed to have improved from her two previous responses (PRT#3 and PRT# 9). In the case of the boys, Max and Ben relied on the same strategies they used before. This time, Max touched the objects with his two hands. Ben, traced the box again, but this time he counted the objects orally up to nine while energetically marking dots inside the traced box. With the exception of Maria, the children seemed quite comfortable and happy with their responses.

Figure 27 Maria's global representation of 14

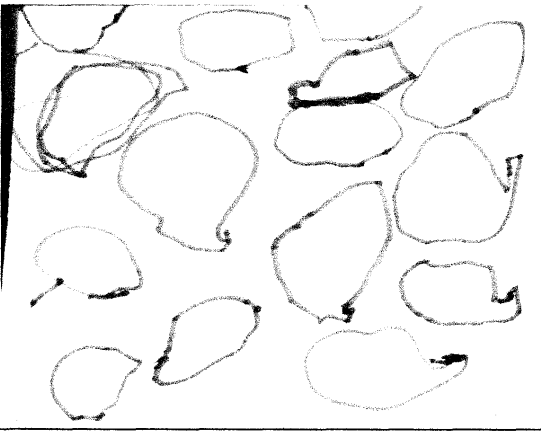


Figure 28 Ben's idiosyncratic representation of 14

4-year- olds PRT# 14

Children interviewed: Cathy, Celia, John and Chad

(a) *Type of representation*: Cathy again used tally marks with no accuracy to the amount. Celia tried the same action she used before: drawing each object in detail. This time though, she became tired and did not complete the task. John traced an idiosyncratic symbol, and Chad printed a number-like-form for "14" while verbalizing "one" and "four" for each of the marks.

(b) *Strategies*: Children seemed to rely on the same types of strategies they used before; however, the larger amount created a sense of frustration in Celia. Before printing she separated the items inside the box and took her time to explore the details. When she started to print, drawing each picture in detail seemed like a long commitment and she decided to stop. She looked tired and overwhelmed. Cathy kept the objects in the box, she looked at them once. Then she traced twelve tally marks.

Chad looked upset this time. He kept looking at the box before printing. He traced the mark on the paper with discomfort. John counted orally up to 14 while touching the objects one- by -one. Then he made two sounds while tracing separately a number-like-form for "1"and a number-like-form for "4." He seemed happy with his answer.

Figure 29 Cathy's one-to-one representation of "14"

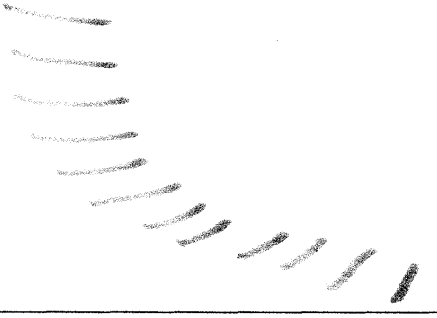


Figure 30 Chad's number-like form representation of "14"

5- year -olds PRT# 14.

Children interviewed: Karle, Heather, Zack and Carlos

(a) *Type of representation*: Karle and Carlos used conventional numerals to represent 14; however, both of them printed 41. Heather printed a cardinal representation. Zack did not print this amount on the paper. He kept looking at me until the time ran out.

(b) *Strategies*: The children used different strategies for this amount. Karle again counted the objects before printing, but this time she moved her mouth whispering the numbers. She stopped at 14, paused to think, and then printed "41." She appeared to be quite happy with her response. Heather counted loudly while touching the objects and after printing "14," she showed a big smile.

Carlos counted the objects by looking at them; when he finished, he stopped, seemed to be thinking, and counted again. He looked at me and said "I don't even know how to spell it." After a few seconds he said; "It is a number I can't spell." He continued to think and did not look happy. When the session was going to end, he decided to take the items outside the box and said "After I count maybe I will know" and counted again. Suddenly he shouted "I know fourteen!" I counted thirteen before. " At the end, he proudly printed 41.

Zack stood up and counted the objects inside the box with the movement of his eyes. He touched the items and said, "I don't know how to make 13." After a few seconds, I asked him if he could write how many in a different way to which he responded "I guess". He looked frustrated and we decided to stop the session.

Figure 31 Heather's cardinal representation of "14"

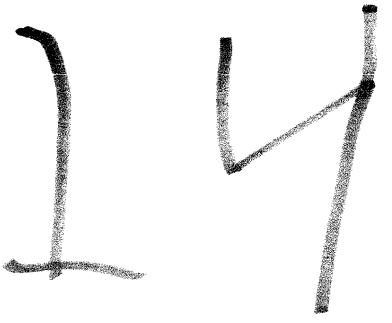
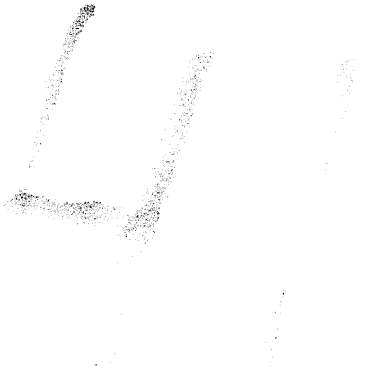


Figure 32 Carlo's conventional representation of "14"



PRT#14 discussion

The 2- year-olds children sustained the same type of printing response for this amount that they did previously (PRT#3 and PRT#9).This age group showed no worries about the types of numeric answer; talking about the objects seemed to be the major interest. Overall, printing was again seen as a fun activity through the use of scribbles.

The 3 – year-old children seemed to understand that the amounts had changed; therefore, the notion that there was an increase of "lots" (from 9 to 14) needed to be written on the paper. It is interesting to observe how, with the exception of Laura who was scribbling, the representation of "lots" appeared to follow a pattern in Ben's, Maria's, and Max's printing: dots-objects-tally marks. This pattern seemed to progress in a sequence that evolved from the fun action of stamping dots to the use of a more advanced form of analogic symbols such as tally marks. Therefore, it could be suggested that this progression in children's types of numeric notations is related not only to their possible understanding of numbers but also to their developmental patterns of drawing and printing.

The strategy of repeatedly referring back to the objects to identify which ones had already been traced was used by Maria and Max. Perhaps because of the value given by these children to each object, every time they checked, they look at one particular object. However, when the amount increased (14), it appeared that they lost track and Maria seemed to experience a sense of frustration in terms of the amount of objects she has to draw. Concentrating on each object is a difficult job (There is a lot to do," said Maria when she looked at the box that contained 14 items). As previously described in my literature review, these children cannot see the idea of a set and merely rely on each

object individually. However, the richness of the responses observed in this group relies on how the children have started to consider the idea of amounts (a quantitative notion) from a qualitative perspective such as drawing details of the objects.

Once again, the responses of the 4-year-olds varied depending on the gender. The girls' representations were global and not accurate. On the other hand, the boys know that they need a conventional symbol to write "14." Knowing how to do this was expressed with pride; not knowing how to print the symbol, seemed to cause frustration.
These children used the strategy of referring back repeatedly to each item while printing. However, in the case of "14," this strategy became more difficult to be used efficiently.

In regards to the 5-year-olds, it appears that all these children seemed comfortable with the idea of numbers; however, they also seemed to understand that higher numbers (like 14) require using other strategies, such as taking the objects outside of the box, using both hands for counting or counting the entire set twice.

An interesting observation regarding the 5 year old boys' responses was that in both cases, they became frustrated when counting number 13 and realizing that they did not know how "13" look and, as a consequence, they could not print it. Number 14 was seen as an easier one to visualize and print. Based on these observations, it appears that these boys seem to make a linkage to the visual representation of the numeral and relied on their knowledge of what the numeral looks like.

PRT discussion

After describing the children's responses and strategies for each amount represented by age, there are some important observations in regard to the PRT that I would like to mention.

The 2- year-olds' responses seemed consistent throughout the task. These children's responses mainly relied on exploring the objects. Their printings do not show numeric ideas.

Overall, when asked to represent "3," children ages 3 to 5 have a certain familiarity with this number. This familiarity includes, for example, knowing what "3" looks like; counting loudly up to three or looking at three objects and responding with pride ("there are three"). Three, as a small number (Piaget, 1962), seemed easy to explore by touching, by looking and when trying to keep track of the objects already represented on the paper.

Some interesting observations arose from the children's representations of 9. From age 3, children seemed to be aware that the amounts have changed in comparison to the previous box. Verbalizations such as "lots" and "there is more" and the representation of more tally marks or pictures, are clear examples of this. It appeared that children know what this numeral looks like from age 4. However, some children did not know how to print this numeral and this seemed to cause frustration. Each of the 4-year-old participants represented "9" in four different ways.

Representing 14 appeared to be related also to the notion that the amount had changed and the children therefore, included more tally marks, dots, and pictures. But also the notion of what "14" looks like seemed to make a real difference in the 5-year-

olds boys' responses. Printing 14 seemed to challenge some of the children's strategies. It was observed that children who responded quickly for the printing of "3" and "9," needed to rely on strategies such as counting with two hands or counting the entire set twice.

Reading Response task (RRT)

Purpose

The purpose of the RRT was to observe and describe in detail how children read the numerical notations they printed during the PRT.

Protocol

Twenty minutes after the PRT was completed, children were asked to return to the room to read the cards they had previously printed. The boxes were removed for this part of the task, not allowing children to rely on visual or spatial clues.

The reading task is partially adapted from Bialystok and Codd (1996) whose study explored the linkages between cardinality and written numbers. By reviewing the video many times, I was confronted with the issue that most of the participants' responses did not completely fit into categories of previous studies. For this reason, I developed, with partial adaptations from Bialystok's and Codd's study (1996) a table that considers the children's stages of number reading:

Table 6 Stages of children's reading

Stage	Description
1- Global reading	The child could: (a) name/talk about the objects (b) read what the printing resembles
2- Reading the objects	The child could: (a) name the objects broadly (b) name object's attributes / values (c) point at objects (d) use descriptions such as "a lot", "no more," "empty" "little"
3- One by one object reading	The child could: (a) names every object (one-by-one) (b) points at tallies/ marks (c) use descriptions such as "ä lot", "no more," "empty" "little".
4- One by one number reading	The child could point at objects and count orally. The child does not considered the last number as the final answer.
5- Cardinal reading	The child understands the final word as the one that indicates the amount in a set.

Pedagogical implications of this table will be discussed in Chapter Five.

When the children came back to the room I told them that when I was putting my things away, all the papers they printed for me got mixed up and that I needed help to know what they wrote on each one. I showed, one at the time, the three copies they did before. To facilitate the reader's understanding, these task responses are described separately as RRT#3, RRT#9, and RRT#14.

Children's responses by amount are summarized in Table 7 (bold text refers to strategies).

Table 7 Summary table for Reading Response Task (RRT)

Age/ Amount	2 year-olds	3 year-olds	4 year-olds	5 year-olds
3 Objects	Two children did not know One reading response was not related to the question One Global reading response **Looking way; no interest**	One child did not know One reading response was not related to the question One reading the object response One one-by-one object reading response **Overall confident about their responses**	Two reading the object responses Two cardinal reading responses **Looking at the paper for a few seconds** **Looking down**	Four cardinal reading responses **Looking at the paper and responded quickly**
9 Objects	One child did not know One reading response is not related to the question One reading the object response **Looking away; did not know the answer**	One child didn't respond One reading response is not related to the question One global reading response One one-by-one object reading **Looking at the paper for a few seconds ;one child seemed nervous** **Other children seemed confident about their responses**	One child did not know One reading the object response One one-by-one object reading response One cardinal reading responses **Certain sense of surprise of not being able to read the printing**	Four cardinal reading responses **No hesitation ; certain sense of "this is easy"**
14 objects	Two children didn't know One reading response is not related to the question One global reading response **Looking away; no interest**	Three reading the object response One one-by-one object reading response **Looking at the paper** **Identification of objects**	Two children did not know One reading the object response One one-by-one number reading response (a 4) **Certain sense of frustration of not being able to read the printing**	Three cardinal reading responses One child did not respond **He seemed tense while moving his eyes from the paper to me.** **The other children seemed confident even though they printed 41**

2- year- olds RRT#3.

Children interviewed: Julia, Gloria, Connor, and Alex

(a) *Type of response*: "I don't know", Julia expressed looking at her first print. Connor responded the same way; Gloria was very quick to say "G", for Gloria."
In the case of the boys, Connor also expressed he did not know what the notation said; Alex, instead looked at the notation and said "a nose."

(b) *Strategies*: The children looked relaxed. They did not seem to have much interest in the activity and looked away without paying much attention to the prints.
Alex respond "a nose" referring to the form like that the scribble has (It did look like a nose). Gloria, after naming the first letter of her name, recalled the amount of objects from the box and said "There are three."

3 –year-olds RRT#3

Children interviewed: Maria, Laura, Ben, and Max

(a) *Type of response*: Maria looked at the paper for a few seconds and gave no answer. Laura responded by saying "G" after looking at the scribble she made.
The boys' responses were quite different from each other. Max pointed at each of the three tally marks he did and said "One here, one here, one here." Ben, who traced the box before, simply responded "a box."

(b) *Strategies*: Except for Maria, who seemed a little bit nervous after looking at the prints she did previously, and did not respond, Laura, Max, and Ben, appeared to be quite confident with their responses. These three children looked at me with pride after reading the notes.

4- year- olds RRT# 3

Children interviewed: Cathy, Celia, John, and Chad.

(a) Type of response: The girls responses were related to qualitative characteristics of the representation and read the objects on the paper; "I did lines," responded Cathy, while Celia named each of the objects she drew with so many details: "A heart, a fish, a sheep." The boys' responses related to the cardinal value of the set. They both responded: "Three."

a) *Strategies*: Cathy did not look happy with her response; her eyes looked down with disappointment. Celia and John responded with confidence after looking at the prints. Chad's response took a few seconds; he looked at the paper, then he moved forward to get a closer look and finally responding.

5 –year- olds RRT# 3

Children interviewed: Karle, Heather, Carlos, and Zack.

(a) *Type of response*: Karle, Heather, Carlos, and Zack responded in reference to the numeral they represented: "Three."

(b) *Strategies*: The girls and Carlos appeared very confident about their responses, possibly because their representations were distinctive in some way. These children smiled and seemed to be having fun. Carlos looked at me and said "I did not write that." Then he said "Oh yes…It's a three." Zack in contrast, responded nervously, and looked at me for approval. He seemed relieved after realizing that he did respond well.

RRT#3 discussion

It was observable that none of the 2-year-old children used numeric responses when reading their printings. The 2-year-old children responded simply by saying "I don't know" or by trying to read based on what the drawing resembled (for example a nose). Overall, based on the strategies that these children used, it appeared that reading seemed to be not as fun and interesting as printing.

At three years of age, it could be suggested that children used, for the first time reading strategies, such as moving their eyes through the paper searching for clues. The printed symbols were identified by some of the children, but they were not related to the amount in the boxes.

Max gave a one-by-one response, and for the first time, quantity was considered. "One here, one here, one here," is a clear example of one-to-one reading and of how the set is seen by him as composed of individual items.

The 4- year-old children read what they saw in the paper: lines, pictures, and numerals. When they were not able to read the printed symbols, they become frustrated or surprised.

Cathy's responses were very interesting. She was able to print using an analogic representation (tally marks), but when she read them, she did not relate the meaning of the tally marks to a quantity; instead, she read "lines." The lines were seen (and read) as objects with no relation to the amounts she intended to represent. As Bialystok and Codd (1996) expressed, it is clear in these kinds of examples that these children are still not able to understand the nature and the intentionality of symbolic representations. In the

case of the 4-year-old boys they confidently responded after reading their own notations, possibly because their printings were closer to conventional ones.

In regards to the 5-year-olds, it was observed that this age group understand the numerical symbols they used to represent the sets they previously explored. In other words, these children were capable of reading a conventional numeral such as "3" that the previously had printed.

RRT#9

2 year olds RRT#9

Children Interviewed: Julia, Gloria, Connor, and Ale

(a) *Type of response*: Julia and Connor responded negatively; "I don't know" replied Julia again. Gloria read "G for Gloria." "A fish," said Alex, after looking at the small scribble he previously traced. There was a fish in the box, and perhaps that is what he recalls when looking at the scribble.

(b) *Strategies*: once again, the children looked away and moved their head in negative motion to express that they did not know the answer. These children were not worried about their responses. In the case of Gloria, a sense of pride was perceived when she looked at me and said, "G for Gloria." Alex was able to read his own printing (a fish). Alex and Gloria were able to accomplish a qualitative reading of the printed responses.

3- year- olds RRT#9

Children interviewed: Maria, Laura, Ben and Max

(a) Type of response: Maria took a few seconds to look at the circles she had previously printed on the paper and gave no answer. Laura responded a "B" after looking at the scribble she made.

Max pointed at each of the tally marks he made and said, "One goes here, one goes here, one goes here." Ben looked at the print and responded "a man."

(b) *Strategies*: Maria's nervousness was expressed by her face. She looked at the paper and seemed to realize that the tracing she did for each of the objects was not clear enough and therefore, difficult to read. Laura, Max, and Ben, appeared to be quite confident with their responses. Again, these three children looked at me with pride after reading the notes.

4-year- olds RRT#9

Children interviewed: Cathy, Celia, John and Chad.

a) *Type of response*: "I did lines," said Cathy. Celia pointed and named only the objects she was capable of recognizing from her drawing. This time, John shrugged his shoulders to express that he did not know. Chad was able to read his own printed numeral: "nine," he said.

b) *Strategies*: Cathy, again appeared to express discomfort with her answer and looked at me for approval. Celia showed surprise that she could not rely on her own drawing for some of the objects she did before. John's confidence seemed to disappear

after not recognizing his own printing. As Cathy did, he looked at me for clues and /or approval. Chad, on the other hand, responded quickly and very confidently.

5- year- olds RRT#9

Children interviewed: Karle, Heather, Carlos, and Zack.

a) *Type of response*: All of the children responded quickly and in reference to the number "nine"

b) *Strategies*: This time Karle, Heather, Carlos, and Zack's responses showed confidence. There was no hesitation and they looked at me with a certain sense of "this is easy."

RRT#9 discussion

The 2- year old responses were again not numeric responses when reading their printing. Some of the reading responses were based on the qualitative aspects of the set. For example, it was interesting to observe how Alex (who read "a fish") appeared to be capable of reading his own printing (perhaps a fish). It could also be suggested that he remembered the fish in the box. In any case, the qualitative aspect of the set seemed to be what really mattered for this child. Overall, the 2-year-old children seemed not interested in the activity.

The same type of response was observed in some of the 3- year- olds, whose readings relied on what these children remembered as being in the box or by reading what the representation resembled ("a man," "a B"), but again, there seemed to not be a

consideration of the amount. Therefore the readings were not accurate from a number perspective. Some children appeared quite confident about their responses.

Each 4-year-old child responded differently in this particular task. The responses ranged from not responding at all to having the capability to read "9."

At 5 years of age, all children were capable of reading their own notations for the number "9." Therefore, it could be suggested that the numerical symbols they used represent the set of nine they previously printed

RRT#14

2- year -olds RRT#14

Children interviewed: Julia, Gloria, Connor, and Alex

a) *Type of response*: Julia expressed, once more, "I don't know"; Connor also stated that he did not know. Gloria read "G for Gloria." "Another fish" said Alex, after looking at the paper.

b) *Strategies*: children looked away or, like in the case of Connor, moved their shoulders up and down to express that they did not know the answer. Gloria smiled after reading "G" once more, and Alex seemed quite confident with his response.

3 year olds RRT#14

Children interviewed: Maria, Laura, Ben, and Max

a) *Type of response*: Maria looked at the paper again; this time she said "a bat."

Laura responded "an eight" after looking at the scribble she made. Max pointed at each of the tally marks he made and said, "One goes here, one goes here, one goes here." Ben looked at the print and responded "people."

b) *Strategies*: Maria searched for clues in the paper while moving her eyes through the tracing she previously did. She seemed relaxed when she was able to identify the tracing of one of the objects in the box: a bat. Laura, Max, and Ben responded using the same strategies they used before: looking at the paper and responding with confidence.

4- year- olds RRT# 14

Children interviewed: Cathy, Celia, John and Chad

a) *Type of response* "I did more lines," said Cathy. Celia looked at all the objects she drew before and responded "I don't know." This time the boys' responses changed from their previous one. John's response related to one of the digits in the number "four." Chad's response was "I don't know."

b) *Strategies*: Cathy looked at me as justifying her previous representation. Celia looked at the drawing again and responded "I don't know." This time she looked upset and tired. John's response was quick and confident while Chad seemed frustrated with not being able to read his own printing.

5- year- olds RRT# 14

Children interviewed: Karle, Heather, Carlos and Zack.

a) *Type of response*: Karle, Heather, and Carlos responded loudly "fourteen."

b) *Strategies*: While Karle, Heather, and Carlos responded with no hesitation, it is

interesting to observe that two of these children, Heather and Carlos, did print a 41. They seemed not to realize that and the responses were quite confident.

RRT#14 discussion

The 2 -year-olds responses seemed not to change from one amount to the other; a sense of no interest in the activity is once again expressed by most of the children's body language. Children relied on the same types of responses they used before when presented with the printing notation they previously did.

Once again, the value that the objects played in this task was observed in Maria's response. While moving her eyes along the paper, she was able to identify the trace she previously did of a "bat." Her reading was supported by this finding. This object seemed to be seen (and read) as an object itself, with no relation to the total set of 14; therefore this particular example could be showing how, for some children, the idea of a set is still quite fragile because each item is seen individually.

At 5 years of age, the responses to the reading task, with the exception of Zack's, appeared to be consistent and accurate. In their reading responses Karle, Heather and Carlos read the numeral 14 alone with no reference to the objects. It was interesting to observe how, even though two children printed the numeral 41, they were still able to respond accurately "14."

RRT discussion

After reviewing the video tape repeatedly, it appeared that the reading task seemed to be not as interesting and fun for the children as the printing task was. Perhaps the possibility to explore and play with the three boxes was perceived as being more fun than reading their own notations.

The 2- year- olds' responses seem consistent throughout the task. Reading strategies such as looking at the clues, was completed briefly and with no interest.

When children were asked to read their printing notations for "3," cardinal responses (the possibility to read "three") were accomplished by almost all of the older children (four and five year olds).

In regards to reading the notations for "9," the 3-year-olds read what they saw on the paper, for example lines. They also read what the drawing resembled: "a man" or a "B."

The types of responses when reading of numeric notations for "9" seemed to change slightly, particularly for the 4-year-olds. Each of these four participants responded differently (one child did not know, one reading the object response, one one-by-one object reading response, and one cardinal reading response). It was also interesting to observe that a certain sense of surprise was expressed by one girl for not being able to recognize what she wrote.

The 3 -year -olds and the 4 -year -olds read the numeric notations for "14" with almost the same type of reading responses they did previously for "9."

Most of the 5- year -olds were able to accomplish the cardinal reading of the amounts "9" and "14." It could be suggested that the 5 year olds responses seem to be

consistent and accurate in these tasks. These children were, most of the time, able to rely on the meaning of their own conventional written notations.

CHAPTER FIVE: DISCUSSION

Chapter Five is a discussion centred at the intersections of the findings in this study and the theoretical frameworks considered in the literature review. I will examine each of the tasks the children completed independently and place the discussion into the context of current early childhood mathematics pedagogy while challenging some of the current practices and beliefs. Several emerging themes were evident in the data analysis and their implication for teaching and further research will also be discussed.

Introduction

The purpose of this qualitative study was to start the process of creating a pedagogical framework for the teaching of number representation in early childhood classrooms. Based on both existing literature and on the strategies and responses used by sixteen 2-, 3-, 4-, and 5-year-olds, the pedagogical framework may serve to demonstrate when it is appropriate to introduce written numeric representations, as well as what indicators should be considered by early childhood educators (ECEs) when engaging young children with numbers and numerals.

The study explored the following questions: (1) What symbolic responses do young children choose when prompted to represent numbers? (2) What strategies do young children use when prompted to represent numbers? (3) What numeric notations do young children use when asked to print numbers? (4) What are the developmental trends of young children's numeric notations? (5) What strategies do children use while printing numeric notations? (6) What meaning do these notations have for young children? and (7) How could early childhood educators support children's developmental trends and

sequence of strategies used in number representation?

Children from two Early Learning Centres (ELCs) from PEI participated in the study. A total of 47 children were videotaped in whole-class sessions where they had to complete two different tasks: a counting task (Task 1) and a number search task (Task 2). A total of 16 children (a boy and a girl from each age group) were also videotaped in individual sessions where they responded to three different tasks: 1) A Symbolic Response Task (SRT); 2) A Printing Response Task (PRT); and 3) A Reading Response Task (RRT). The purpose of the SRT was to observe children's strategies and responses when asked to use any type of symbolic representation other than oral language. The PRT focused on exploring children's numeric representation after being presented with three boxes that contained different amounts (3, 9, and 14). The purpose of the RRT was to observe children's reading responses of their own numeric notations.

A series of themes emerged for teaching consideration, such as the children's understanding of the sequence of counting, the children's one-by-one understanding of sets, and the children's familiarity with the concept of "3." The use of fingers and the qualitative descriptions children gave of the sets appeared also as pedagogically valuable. These findings will be discussed and placed in the context of implications for early childhood mathematical education, particularly around cardinality, dual representation, the impact of fine motor development, and the influence of children's emotional engagement when completing mathematical tasks. It is important to acknowledge that even fine motor development and emotional engagement were not research questions, both areas "stood out" when analyzing the data.

In addition, the study attempted to develop a better understanding of the value of documenting children's mathematical ideas to support developmentally appropriate early childhood mathematical practices.

Cardinality and dual representation

Writing and reading numerals implies understanding the value a numeric symbol has (cardinality), as well as what meaning that symbol conveys (dual representation) for a social group. Previous studies described cardinality (Bermejo, 1996; Bermejo; Fluck, Linnel & Holgate, 2005; Morales, & Garcia de Osuna, 2004; Zhou & Wang, 2004) as a major principle for the understanding of number and numerals. At the same time, dual representation (DeLoache, 2004; DeLoache, 1995a, 1995b; DeLoache & Burns, 1994; DeLoache & Marzolf, 1992; DeLoache, Miller & Rosengren, 1997) is described as a crucial milestone for symbolic understanding. Based on these previous studies, it appeared that both cardinality and dual representation should be key components of early childhood mathematics education (ECME).

Children's development of dual representation, the ability to mentally represent the meaning numerals have (i.e., printing II for "2," reading "two" for "2"), can be facilitated if cardinality is considered as a main outcome of ECME. Therefore, by proposing activities that support children's development of the cardinal concept, the progressive construction of what numeric symbols mean or represent could also be facilitated. However, it is crucial to understand that both cardinality and dual representation develop gradually during the first years of life. As a consequence, early childhood educators should be aware of how children construct the numeric and the symbolic domains. The following sections of this

chapter discuss main themes that emerged from the children's responses when prompted about numbers and numerals.

Numbers...What do they mean?

Whole class session. Task 1

Emerging themes

Using number words.

A number is an abstract entity (McColskey, 1992) that according to Piaget (1952; 1983) is mentally constructed through developmental stages. Abstracting what numbers mean is to be able to understand the cardinal value numbers have. As described before, the cardinal value is what the number refers to; for example, "4 dolls" refers to "1, 2, 3, and 4 dolls." The value is related to the last word and to the entire group of objects.

The responses my participants gave to the whole class Task 1 session show that these children were able to name the words for numbers such as "there are three," "1, 2, 3," or "I can count to 100." The Task 1 questions, "Who can count? Can you count? Are there any other ways you can count?" were understood by most children ages 2 to 5. Their responses were expressed by using words that referred to numbers. The use of these number words appeared as a crucial symbolic response to be considered from a pedagogical perspective.

Rote counting.

Rote counting implies the use of the words in a sequence ("one, two, three," etc.). The children seemed to know about this sequence, and in most cases, they used it

combining the words with body movements. In some cases children appeared to sing the sequence while moving their bodies in a particular rhythm.

Rote counting was a strategy that children not only used when ask to count (Task 1), but it was also used spontaneously by some children ages 3 to 5, when exploring the three boxes for the Printing Reading Task (PRT). Responses such as "after I count, maybe I will know," demonstrate that rote counting was considered by those children as a tool that helped them to solve the mathematical problem of knowing how many items were in the box. Pedagogically, this type of response could demonstrate that children understand what numbers are for, and therefore activities that support the use of rote counting as a tool to give or obtain mathematical information should be encouraged.

A sequence is based on core repetitions (for example, the 10 number digits for the counting system). With their responses, children ages 3 to 5 showed that they knew that the core repetition had to continue by mumbling or making sounds, particularly for the middle words (i.e., "six, seven, eight"). The counting sequence involves the idea of ordinality (each number has a position in the system) which, as Piaget (1983) explains, is related to the cardinal value each number has. Therefore, if children know that counting numbers implies following a sequence that has a particular order, each time they name a number (i.e., "4") the previous named numbers (i.e., "1," "2," and "3") are included. Children's understanding of the concept of sequence could be related with the "stable order principle" which is described by Siegler (2003), as "saying the words in a constant order" (p.220).

Based on the responses of the participants in this study, naming the numbers from one to ten appeared to develop progressively. As Table 1 (Chapter Four) shows, when

children were asked to count up to "10," the responses varied from not responding at all (2-year-olds), to responding inconsistently (3-year-olds), to mostly responding (4-year-olds), to consistently responding (5-year-olds)in a whole group setting. These responses indicated that the participants' knowledge of all the names in the sequence from one to ten increased with age.

One important consideration should be given to the knowledge most children (ages three, four and five) seem to have about the beginning of the sequence ("1, 2, 3"). Counting up to "3" appeared to be a skill that most children accomplish well. Overall, most of the interviewed children used the three words ("one," "two," "three") with pride and confidence.

Implications for teaching

Previous studies (Bermejo, 1996; Byalistock, 2000; DeLoache & Burns 1994; DeLoache, 2004; DeLoache & Marzolf, 1992; DeLoache, Mendoza, & Anderson, 1999;) suggest that young children name the words for the numbers without understanding the cardinal value and the meaning those words have. Therefore, according to these studies, dual representation (i.e., abstracting the idea of "2" by saying "two") is quite fragile at young ages.

From a pedagogical perspective, the findings of this study support the idea that even if the understanding of the cardinal meaning is still fragile, familiarity with numbers and with language related to numbers is an important outcome to be considered by those teaching early childhood mathematics education (ECME).

Counting is a common practice routine that children do in early childhood settings; (for example children are required to count when waiting in line, when sitting on a circle, when singing or in the gym). Through the Task 1 session, most boys and girls appeared to enjoy the activity of counting with their classmates. However, from a pedagogical perspective, it appeared that in most of those situations the purpose of asking children to count is not clear. A study conducted by Lee and Ginsburg (2007) described that in most scenarios, early childhood educators did not know why they did simple routines, such as counting. Similar findings have also been suggested by other studies (Graham, Nash & Paul, 1997, as cited in Lee and Ginsburg, 2007; Kamii & Kato, 2005; Layzer, 1993, as cited in Lee and Ginsburg, 2007). The findings of this study suggest that the ability to use number words when asked to count is knowledge that many children have (mostly transmitted by adults). Pedagogy should support the development of this knowledge by supporting children in attaining number meaning. In other words, counting routines should be facilitated with the aim of developing cardinal understanding.

Based on the children's responses, it could be suggested that rote counting should be supported and challenge according to the ability of naming the numbers (Task 1) each age group seemed to have (Table 1). In that sense, activities for counting could become meaningful and constructive for young children. In addition, special consideration should be given to activities that imply counting sets no larger than three. In this case, early childhood educators could be supporting number construction and the development of cardinal meaning by emphasizing the perceptual characteristics "3" seems to have. In other words, by rote counting up to "3," only "1" and "2" are included in the total group and it appears that children could easily refer to and represent this amount. The idea of

working with small sets is also supported by researchers such as Teubal and Dockrell (2005) who explained that small numbers facilitate the accuracy of the responses. Responding accurately could certainly support positive reinforcement and emotional engagement when educators provide mathematical activities for young children. The value that small numbers seem to have for ECME will be extended in the description of how children manipulated three objects during the Printing Response Task (PRT), as well of how sets of "3"were printed.

Another particular consideration refers to the way some of the children (mostly ages 4 and 5) loudly named the last two words in the sequence ("nine, ten"). As previously explained, Siegler (2003) suggested that these children have started to consider the value of the last word as referring to a set, and this is an important step towards the understanding of cardinality. Therefore, mathematical activities that encourage children to emphasize the last words in counting sets could be proposed, particularly to children ages 4 and 5.

In conclusion, the findings of this study suggest that while the routine of rote counting could be socially transmitted from adults to children and in most cases learned by memory (Byalistock, 2000), children are continuously being exposed to it, and therefore, use it often. From a pedagogical perspective, and based on the children's responses, counting up to "3" appears to be more meaningful and an important frame to consider for facilitating number construction and the development of cardinal understanding.

Symbolic responses...no words allowed

Individual session- SRT

Emerging theme

The use of fingers.

This study supports DeLoache's (2004) definition of symbolic as any form of response used by children that implies the use of a symbol to represent a quantity (i.e., words, use of manipulatives, or printing).

During the Task 1, children represented numbers using name words through rote counting. In the Symbolic Response Task (SRT), when words were not allowed, the symbolic response that most children chose was the use of their fingers.

The aim of this task was to observe how children answered the question, "How old are you?" without using numeric words. In other words, I wanted to explore the symbolic responses children choose to represent their ages. Half of the interviewed children were intentionally able to represent their age by accurately using their fingers. Most of the finger responses were adopted by children ages 4 and 5.

Representing with words involves using one symbol (the word that represent all the objects in a set) to communicate meaning. Printing offers the same possibility; however, none of the interviewed children relied on the printing form even when markers and paper were at hand. Instead, the children choose to use their fingers as an analogic representation. According to Bialystock and Codd (1996) "analogue representations are a legitimate and popular system for representing quantity" (p. 289). It is interesting to note, that even though some of the children used conventional symbols when asked to write numbers (Printing Response Task) they chose a one-by-one response in the SRT.

This is the case in some of the responses of children like Chad (4), Karle (5), Heather (5), and Carlos (5).

The tendency to use fingers as a symbolic response is related to the use of analogical representations such as tally marks. According to Hughes (1986), there could be a certain connection between these analogue systems (tally marks) and children's fingers. Furthermore, Hughes (1986) and Sinclair (1991) refer to these kinds of representations as a very powerful way to represent quantity even through adulthood. It is interesting to mention that the mathematical term "digit" comes from the ancient Latin *digita* which means fingers. This approach is rooted in the idea that the 10 digits of the hands correspond to the 10 symbols of the common base 10 number system. As previously described, the children were able to consider and use their fingers as an efficient strategy to represent their age.

The children were able to meaningfully communicate their age in the interview. The one-to-one response demonstrated an intention to communicate (i.e., showing three fingers if the child was 3-years-old), and the intention was easily understood by others (in this particular case, the researcher). As DeLoache (2004) expressed, intentionality is a characteristic of symbols, which in this particular scenario, children accomplish accurately.

Implications for teaching

There seems to be an emotional component in the responses to the question "How old are you?" Supporting children to make numeric connections should be rooted in what emotionally makes meaning for them. Furthermore, numeric connections should never be

taught in isolation and in what is not relevant to young children. In terms of emotional involvement, the findings revealed a different level of engagement between boys and girls in relation to this activity. The girls seemed to be more enthusiastic when playing the game of not using words and overall responded with confidence. Some of the boys appeared unsure about their own answers, and, for example, showed their fingers with a certain level of insecurity.

Previous studies have mentioned (Bermejo, 1996; Bermejo, Morales, & Garcia de Osuna, 2004) that it is very difficult to determine children's knowledge of cardinality just because children understand certain structures of the number system and are familiar with them (i.e., rote counting). Overall, this understanding is grounded in the children's social contexts. Perhaps the use of fingers to represent age could also be socially transmitted information.

Most of the older children (4-year-olds and 5-year-olds) responded accurately by using fingers; this could show that these children have an understanding of how sets are composed: one-by-one. Once more, the value of considering small sets facilitated the accuracy of the responses (Teubal & Dockrell, 2005). In fact, all children who responded by using fingers or fingers and words, responded accurately (i.e., three fingers if that child was 3 years old). The ability to respond accurately by using fingers has a special value based on the children's ages (no responses implied more that 5). From a pedagogical perspective the ability to respond accurately by using fingers could be considered when designing counting activities. By challenging young children to make connections between the representations of small sets (no more than five items) with their

fingers, ECME could be developing an interesting approach for number representation that does not necessarily need to rely on the use of paper and pen.

It is important to emphasize that the analogic responses some of the children gave (particularly at ages 4 and 5) show the ways these children think about sets: one-by-one. ECME activities should consider this knowledge as the level of understanding children have (Vygotsky, 1962), knowledge that is also related to the one-by-one counting sequence with which children also appeared to be familiar. Perhaps, this way of thinking of sets (one-by-one) should be considered by ECME as a milestone to accomplish during the early years, rather than a response that is still incomplete. The findings of the study call for a revision of the traditional pedagogical push for conventional printing (use of one symbol), when it appears that these children are considering sets in a one-to-one relation.

In summary, two main themes for pedagogical consideration emerged from children's responses on the SRT: (1) by not using language, one-to-one appears as the way most children, ages three to five, represent an amount and (2) by not using language, it was very interesting to reflect on the valuable role that number words appear to have; first, as way to support, through rote counting, the one-by-one understanding children seemed to have; and second, to synthesize by using one word, the total number of objects in a set (cardinality). In this sense, even if counting is associated with memorized oral routines (Byalistock, 2000), the role of oral language appeared as crucial for teaching and supporting the development of number meaning. Language, as expressed by Anderson, Anderson and Shapiro (2005), facilitates children using mathematical discourse.

According to these researchers, mathematical talk appeared to be "significant in children's mathematical development" (p. 21).

Based on the findings and conclusions already described from the Task 1 and from the SRT, it is critical to reflect on the outcomes to be considered by those teaching early childhood mathematics education when young children are asked to count. The ability to rote count, so commonly known as a basic skill in the field of early childhood, shows that our young children know about sequence and about naming number words and that they are developing number ideas through a one-to-one understanding of sets. In that sense, rote counting should be neither a teaching, nor an ECME goal. Instead, it should be considered as an understanding young children have (even if it is fragile), that with proper scaffolding, could support the development of cardinal understanding as well as the process of number construction.

Could you write how many?

Individual session: Printing Response Task (PRT)

The children's numeric notation findings are based on the developmental stages described in Table 4, (Chapter Four). The table was partially adapted from previous studies (Hughes, 1998; Sinclair, Siegris, & Sinclair, 1983); I created the table for specific pedagogical purposes. As an early childhood educator, I needed to describe in detail children's ways of printing. The consideration of these details (not generally considered in the previous studies) is crucial for understanding children's level of development and for accomplishing my teaching role when scaffolding learning for my young students. I

considered each of these stages as pedagogically valuable; in each of them, children demonstrated their knowledge and capabilities. In other words, I understand each stage as being valuable, and not as a negative response when compared with the capability to accomplish conventional number printing.

Overall, the types of responses children gave followed the patterns already described by researchers such as Bialystock and Codd (1996), Hughes (1986), Kato, Kamii, Ozaki, and Nagahiro, (2002), and Sinclair, Siegris, and Sinclair (1983). In that sense, children's ability to print from idiosyncratic representations to the printing of conventional numbers increased with age. Based on the participants' ages, some of the main trends that emerged from the observations are summarized below:

- The 2 year-olds responses were mostly idiosyncratic. Hughes (1986) described these representations as ambiguous and with no relation to quantity. The scribbles were large and all over the paper; grasping was rudimentary. In terms of strategies, the oral description of the objects appeared as a common response from girls and from boys.
- The 3 year-olds seemed to mostly rely on one-by-one representations. These representations mostly appeared when the amount increased ("9" and "14") and did not have accuracy in consideration. This age group seemed to understand that the amount has changed. In terms of strategies, girls seemed more reluctant to describe the objects orally and through printing than the boys. Grasping appeared fragile.
- The 4 year-old boys were the first ones who started to consider the intention to print a numeral for each of the amounts ("3," "9," and "14"), by using one symbol (number-like forms). The change in the amount seemed to have an impact on some of

these children's confidence when printing. Oral counting and checking back at objects were strategies used by most children when amounts changed. Grasping has improved and children seemed to hold markers in the most comfortable position.

- Most of the 5 year old participants used number-like forms or conventional printing when printing any of the three amounts ("3," "9," and "14"). However, what did change were the different strategies children used when confronted with the larger amount (i.e., counting by touching the objects, using both hands to count, counting many times). Knowing what numbers looked like appeared to be important for quality in the printing. Grasping was well developed in all participants.

Emerging themes

Representing different amounts. The value of representing "3."

Overall, the responses and strategies of children ages 3, 4, and 5 were very similar when representing "3" and "9." But the strategies, and some of the responses, changed with the printing of the double digit "14". The double digit was represented as "lots" from more 3 and 4 year olds; for the 5 year olds, printing two numerals was mostly related to remembering what the number looked like. Double digits appeared to create a conflict where two numerals were needed to mean one quantity; for example, expressions like Carlos (when talking about 13), "I don't even know how to spell it," clearly showed that the number was understood in terms of one-by-one. The same approach was considered by John who after printing "14," made two separate sounds, ("one" and "four") for each of the marks.

How numerals look.

The "thinking position" that some boys and girls adopted as well as the action of taking a few seconds before starting to print, could reveal that they were perhaps recalling the numeral resemblance. How the number looked seemed to affect the quality of the children's printing (i.e., the direction of the number three half circles), and particularly, the confidence in their responses. For example, expressions such as "I don't know how to make the right number," or emotions such as frustrations and discomfort were expressed, especially by 4 and 5 year olds.

Describing sets.

Describing the sets' characteristics (the objects) was an oral strategy and also a way of printing that some children used. The responses appeared to follow a pattern that is described as follows:

- At 2 years of age, most children name and describe each or most of the objects that were in the set. Children did not print about the objects; instead they talked about them.
- At 3 years of age, the objects were named, described and in some cases, printed (by copying one-by- one, by tracing one-by-one, or by tally marks). These descriptions were mainly completed by the girls.
- At 4 years of age, some of the children needed to name and describe the objects. It appeared that the same children represented the objects one-by-one.
- At 5 years of age, children did not rely on objects' characteristics to represent the three different sets.

As explained in methods in Chapter Three, the boxes that were introduced in the Printing Response Task (PRT) had multiple objects. The introduction of the boxes captured the children's attention, as well as the level of interest and engagement with the activity.

Paper and pen for representation.

Children's numeric notations develop gradually and appeared to follow a pattern of responses that in this study were related to the participants' ages. The same patterns of responses were previously described by researchers like Bilaystock and Codd (1996), Hughes (1986) Kato, Kamii, Ozaki, and Nagahiro (2002), Sinclair, Siegris, and Sinclair (1983, as cited in Bialystok & Codd 1996).

Previous studies (Bialystok & Codd, 1996; Hughes, 1986; Kato, Kamii & Nagahiro, 2002; Teubal & Dockrell, 2005) demonstrated how symbolic development and numeric development seem to interact and complement each other when children were asked to write numbers. Based on the children's responses, the findings revealed that fine motor development also plays a key role in number printing, and therefore, should be also considered when children are prompted to write numbers.

Grasping appeared to improve with age and children appeared more comfortable holding markers as they became older. When children were asked to represent how many items were in each box, the possibility of representing a numeric notation appeared to be influenced by the grasping abilities each child had. It could be suggested that in some cases, particularly between ages 3 and 4, the type of response is related to grasping abilities and might affect how a child could print numeric notations. For example, if grasping is still rudimentary, dots (as stamping marks) could become the way to represent

(see Ben, 3 years old); if grasping is more developed and the child can trace straight lines, perhaps tally marks will be used for numeric representations.

Implications for teaching

Based on my teaching experience, number printing is sometimes pushed and conventional printing appears as an academic outcome that should be accomplished at young ages. However, as previously explained, children write numbers in their own way. As a researcher, observing the sequence of responses children gave by age was a fascinating journey. I tried not only to observe the types of responses they printed, but also the strategies they considered, as well as the emotions that arose during the tasks. In this sense, the observation of children's mathematical responses was framed under a socio-cultural perspective of mathematical learning (Anderson, Anderson & Shapiro, 2003), where the role of language, emotions, body language, and cognitive knowledge were equally valued. Based on those observations, the emerging themes and their implications for pedagogy are discussed below.

In terms of the different amounts represented, the findings revealed that particular consideration should be given to the representation of "3." The familiarity and confidence most children (ages 3, 4, and 5) expressed about rote counting up to three, was also observed when exploring and writing about three objects. For the children who represented "3" through analogic marks (i.e., tally marks), a sense of pride and accomplishment was clearly observed. It was interesting to observe that most of the interviewed children seemed to know what 3 looked like. This knowledge had an impact on the responses, and those children intended to use one symbol for representing "3." For

example, three of the 3 year old children, who made idiosyncratic representations in all the other responses (mainly scribbles), used a small line to represent "3" that perhaps is related to how they remember the look of 3 (see Figure 3, Chapter Four). Again, even if the meaning of the value of 3 is still fragile, the knowledge children seemed to have about what this numeral looks like should be considered for ECME. If printing is seen as an activity that facilitates representation, and cardinality understanding is the goal, a small number such as "3"could frame activities that aim for meaningful construction of number representation. On the other hand, findings revealed that the introduction of double digits should be carefully analyzed by early childhood educators, particularly if cardinality aims for children to understand that one symbol (one mark) means a whole set.

There is no doubt that the knowledge of how numerals look is affected by visual media that every child encounters in different environments. Numerals are posted in signs, books, keyboards, and in many other places. From an educator's perspective, the presence of visual information in the class environment seems to be valuable in supporting this knowledge. However, based on the participants' responses, it also appears that sometimes the knowledge of how a number looks has an impact on the capability of children to represent numbers in their own way. That was the case of John (4), Carlos (5), and Zack (5) who appeared to be upset and frustrated by not remembering the numeral, or by remembering and not being able to write it.

Defining the use of visual tools about numbers and numerals should be discussed from a pedagogical perspective. Educators, and not only manufacturers of resources, should have a voice in deciding what works better for different children and for different

age groups. This study invites early childhood educators to reflect about this issue by asking if young children are constructing number meaning by remembering what a number looks like, Is the understanding of how a number looks a way to facilitate the development of cardinality by encouraging the use of one symbol, or is it another way of pushing children to use conventional responses when printing?

The oral and the printed descriptions of the objects children made appeared also as an important theme for pedagogical consideration. The presence of multiple objects affected some of the children's strategies and types of responses, particularly when printing larger sets ("9"and "14"). Some of the positive outcomes that occurred when children were presented with a variety of objects included: (1) when representing larger sets, the different objects facilitated the children's strategy of tracking which object was already represented, (2) when representing larger sets, the different objects facilitated the children's strategy of counting by looking, and (3) when representing a larger set, accuracy was accomplished by drawing each object (see Figure 17, Chapter Four). On the other hand, the presence of objects showed some negative outcomes such as (1) when representing larger sets, children could become frustrated and tired of drawing each object. In summary, the description of objects appeared as an important strategy that children ages 3 and 4 used. It was interesting to observe that in general, girls seemed to rely more than the boys on these kinds of strategies.

It could be suggested, that by describing objects (one-by-one) children showed what is important to them and how they understand sets: one-by-one. Qualitative characteristics (objects' values) become crucial for facilitating empirical abstraction (Piaget, 1971). However, as expressed in the literature review, empirical abstraction

could facilitate constructive abstraction (mental relationships between objects) if a supportive and challenging environment scaffolds the possibility for children to establish mathematical relations between objects. These ideas have been previously discussed by researchers like Piaget (1953), Piaget and Inhelder (1983), Kamii and Kato (2005) and Kamii and Rummelsburg (2008), who suggested how mathematical knowledge and physical knowledge (the knowledge of objects) are interrelated. It is clear that during the early years, mathematics should be constructed purposely though the manipulation of small sets of objects. Description of objects' values (i.e., colour, weight, size) and of objects' attributes (i.e., smaller than, darker than) should be encouraged and scaffolded also in terms of sets. In that sense, characteristics such as "more," "less," "empty," "lots," should also be described, and young children should be encourage to represent these characteristics.

In addition, and based on the children's responses, it appeared that once again, language facilitated "talking about" the objects. Language, as expressed by Vygotsky (1968) becomes the vehicle of thought and educators should be aware of it. In that sense, activities that facilitate mathematical talk in early childhood classrooms should be encourage as well as documented. As stated by Anderson, Anderson and Thauberger (2008), "classroom conversation may be co-constructed as children and teachers listen, put their stories in a mathematical context, use children's labelled mathematical drawings and number drawings and elicit explanations from each other about how they solved the problems" (p.124).

The findings here demonstrate that to print numbers, there are crucial areas, symbolic, numeric, and also fine motor, to be considered and assessed before asking

children to print symbols conventionally. In that sense, mathematics is not only related to cognitive areas of development; it appeared that the action of printing numerals in early education settings should be encouraged by respecting children's emotional, cognitive, linguistic, and physical development. This idea is related to Gifford's suggestion about a "holistic approach to early mathematics" (2004, p.7). As Ginsburg and Golberck (2004) stated "Mathematical and scientific learning develops in a social context. One cannot separate the learning of science or mathematics from the child's interest, emotions and peer relations" (p.192).

What does it say?

Whole class session- Task 2

Individual session- Reading Response Task (RRT)

The children who were interviewed seemed to know that numerals are written in the environment. During the whole class session, particularly on Task 2, children were invited to look for numbers in their classrooms. Most of the children pointed at different written labels and it appeared that the knowledge of the number name was mostly an ability possessed by 4 and 5 year old children. There seemed to be a sequence of responses regarding to how children saw numerals in relation to other written texts. The younger groups (the 2 and the 3 year olds) pointed indistinctly to any labels (i.e., words, picture, numbers) while the older children (4 and 5 year olds) started to recognize numerals as being different from words, and in some cases as being different from individual letters.

Children's numeric readings were observed based on the developmental stages described in Table 6 (Chapter Four). The table was partially adapted from a previous

study (Bialystok & Codd 1996). Once again, I created the table for specific pedagogical purposes. As an early childhood educator, I needed to describe in details children's ways of reading numeric notations. As expressed before, observation of those details are crucial to support and to scaffold my students' own levels of understanding. As stated in relation to the printing stages, I understand each stage of reading as valuable itself and not only as a negative response when compared with the capability of accomplishing cardinal number reading.

Overall, children seemed to enjoy printing more than reading. In fact, only the children who were able to read conventionally appeared engaged with the activity.

Based on the participants' ages, some of the main trends that emerged from the observations are summarized below:

- Mostly, the 2 –year- olds did not know what the notations said on the paper, or read the notations globally (looking at the paper and pointing at the traces or talking about some objects that they remembered being in the box). In terms of strategies, most children looked away after looking at the paper for a few seconds.
- At 3 years of age, most of the readings were based on what children could "see" on the paper (i.e., pictures, tally marks) or what the drawing seemed to resemble. In some cases, the reading appeared to be related to the objects that they remembered in the box.
- The 4- year- old boys who printed number-like forms (Chad and John) were able to read their own notations. The girls, who used analogic representations, read what they saw on the paper (i.e., tally marks, pictures) without consideration of reading numerically. This observation was also described by Byalistock and Codd (1996),

who stated that young children cannot see the "analogue display as both as a drawing and as a symbol for quantity" (p. 289). These observations showed that young children consider symbols (even tally marks) without numeric meaning.

- Most of the 5- year-old participants used number-like forms or conventional printing when printing any of the three amounts ("3," "9,"and "14"). Most of these children were able to read their notations. It could be suggested that numerals facilitated accurate reading and that at this age, the numerals represented did have meaning for most children. It appeared that reading is completed in a one-by-one relationship; therefore, the reading of a double digit (that was printed as "41") was competed in a one-by- one motion: "four-teen."

Emerging themes

As previously explained, children write numbers in their own way. Overall, it appeared that children's numeric representations are not reliable for reading numeric meaning. Previous studies (Byalistock, 2000; Byalistock & Codd, 1996) have explained that cardinality is not achieved at young ages, and that the printed symbols, even when printed one-by-one, have no numeric meaning for young children. Overall, and particularly for ages 3 and 4, the meaning appeared to be related to qualitative characteristics (object drawings, and object resemblances) while at 5 years of age, the children appeared to understand the numeric symbolic meaning their notations have.

Several themes emerged from the children's responses that should be given pedagogical consideration. These themes and their implications for ECME are discussed in the following section.

Ages and stages.

It appeared that accuracy in the reading responses, started to become more frequent as children got older, particularly at age 5. The reasons for accurate responses appeared to be related to the understanding many older children have about the conventional meaning of symbols. Conventional meaning implies that when printed, others can understand and read the symbol. Because young children have their own ways to represent numbers, and the tracing quality differs from age-to-age, the printed symbols do not communicate numeric meaning.

On the same hand, and supported by this study's findings, these children (in particular ages 3 and 4) seemed to understand sets one-by-one. Thus, when reading sets, pointing at tally marks one-by-one or expressions such as Max's "One goes here, one goes here, and one goes here" become the strategy most children appeared to use. However, in some cases, the one-by-one reading became inefficient, because the printing was not clear enough and the children could not distinguish the traces or figures they made on the paper.

Emotional engagement.

Educators know that individuals learn best when they are emotionally engaged. Mathematics activities that work against children's levels of engagement could hardly become meaningful. As Gifford (2005) states, strong emotions are often involved when learning mathematics.

As expressed previously, the level of engagement of children who did not use number-like forms or conventional printing was limited during this task. During the Reading Response task (RRT), children's body language showed more emotional responses than in any other task. In most cases, children ages 3 and 4 appeared to be

frustrated, confused, or even surprised at not being able to read previous notations. These children, who had previously represented sets and who had been actively engaged in the Printing Response Task (PRT), suddenly appeared worried and upset.

It is important to consider, based on the study's findings, that some of these children have certain ideas of how specific numbers look, which could be influencing their responses when they do not see that idea represented on the paper.

Implications for teaching

Reading numerals is related to the capability of reading one symbol for example, "9" by using one number word, "nine." This capability implies understanding the cardinal value numbers have (Bermejo, 1996; Bermejo, Morales, & Garcia de Osuna, 2004) as well as what the numeral represent (dual representation).

Overall, it appeared that reading was more challenging for young children than printing numeric ideas. While printing facilitates the representation of personal ideas about quantity, reading relies on shared understandings social groups have. The findings suggest that cardinality and the capability of conveying meaning (dual representation) from the printed symbols, showed a crucial stage of development between ages 3 and 4 of which educators should be aware. Overall, the analogical representations did not convey any numeric meaning for these children and a sense of frustration was clearly expressed. Early childhood educators should be aware of these stages in development, otherwise, as Ginsburg and Golberck (2004) proposed, there exists the risk of "of pushing young children to learn concepts beyond their cognitive limits" (p.192). From a pedagogical point of view, it is crucial to understand that before children can print number-like forms or

conventional numerals meaningfully (that is with an intention and a desire to communicate quantity), reading numbers appeared to not be a developmentally appropriate activity. The findings of this study suggest that early childhood educators need to reflect about the challenge that questions such as "could you tell me what you wrote?" or "could you tell me what number is that?", could pose to some 3 or 4 year olds if they are printing numbers in a one-to-one relationship (i.e., by using tally marks or pictures).

Limitations of the study

During the process of data analysis and discussions, some limitations appeared as important considerations and are shared here as possible limitations to this study:

1. A small sample size was chosen for this study. The study's findings represent the responses from a small sample of children who live in urban areas and who attend early learning centres with similar socio-economic characteristics. In this sense, only one socio-economic status could be represented in the participants' responses.
2. The children who participated in the individual sessions were chosen by the early childhood educators. During that selection process, educators were asked to select children who exhibit typical behaviours for their age group. Since, no formal assessment tools were used to determine participants actual match to expectations for their age group, it could be questioned how well this group was a representative population. As well, the findings of the study do not show responses from children who may have different learning abilities.

Considerations for future research

With limitations identified in the sample chosen for this study and the number of participants included in the samples, it could be suggested that there is a need for further research in early childhood teaching practices drawing from larger and more diverse populations. As well since only one major area of mathematics development was explored, it could be said that more research is needed in different areas of early childhood mathematics. Some of the recommendations for future research are:

1. A larger number of participants, as well as a broader socio-cultural population, for example urban and rural, could be considered for extending this study's findings.
2. A richer picture could perhaps be constructed by exploring and documenting the ways children print numbers in their immediate environments, particularly in early learning centres and at home. The way number printing develops when it occurs with others, peers and adults, could become an interesting area for future research.
3. Exploring the impact that documenting children's numeric responses could have in early childhood programming and on teacher's understanding of how children think numbers and numerals, could also be considered for future educational and/or mathematical research.

Early childhood mathematics education...for young children

Summary

The meanings of numeric symbols are part of a socio-cultural heritage. According to Geary (1995, as cited in Bialystock & Codd, 2000), the meaning of conventional symbols is something that children need to learn. Adults play a valuable role in teaching

and sharing this knowledge. Under this framework, early childhood educators' role in supporting children understanding of numbers and numerals is crucial.

From a socio-cultural constructivist perspective, educators provide scaffolds for children's learning processes, as well as facilitate children's processes of enculturation (which involves, for example, understanding the conventional meaning of numerals). Scaffolding and guiding children through the process of number construction and number representation requires an adult with a solid understanding of the mathematical processes children generally follow. As expressed previously, knowing this sequence and knowing what children can do is crucial for early childhood pedagogical practices. The findings illustrate that consideration of the thinking process of young children is a must for early childhood educators.

This study also revealed the importance of understanding children's development by documenting their numeric responses. Previous studies findings about children's developmental stages when printing numbers (Bialystok and Codd, 1996; Hughes, 1998; Sinclair, Siegris, & Sinclair, 1983, as cited in Teubal & Dockrell 2005) could be considered for this purpose by early childhood educators.

Documenting children's writing development is a common practice in early childhood literacy. The documentation of children's printed numeric notations should also become a rich source of information for educators' scaffolding role. Based on the study's findings, the documentation of children's printed numeric notations could facilitate educators:

(1) developing an understanding of how each of his/her students think about numbers and numerals;

(2) developing a solid teaching approach for scaffolding a child's current level of numeric understanding;

(3) elaborating developmentally appropriate children's mathematical assessments; and

(4) enriching developmentally appropriate mathematical activities regarding numbers and numerals.

To summarize, the findings from this study were constructed to facilitate discussion and to challenge early childhood mathematical education practices. It was my intent to build a space to share and reflect on the best ways to teach number printing to our young students. The edited video which I expect to create for my data will become the main source to inspire further discussion. Through the video, early childhood educators will be able to observe and discuss children's ways of understanding number printing and children's processes of construction, as well as the different mathematical domains involved in children's processes of number construction.

This study has shown that children's numeric notations do not rely on conventional printing between ages 2 and 4. Therefore, early childhood mathematical practices cannot be supported by approaches that aim for production of conventional printing and reading of numbers only, especially during a time in life when key domains (mathematical, symbolic, and fine motor) are in a stage of development.

This study reveals that the different symbolic responses that children use to represent numeric ideas are the main source of information about how children understand numbers. Maria, like other young children, was able to count enthusiastically by herself and with friends; to describe sets by using words such as "more", empty" and "many"; to represent "how many" by using fingers, pictures, and tally marks. The

observation, documentation, and most of all, the respect for these representational responses, are a must for meaningful and developmentally appropriate early childhood mathematical practices.

Epilogue

... *The educator came closer. She wanted to see the pictures of the lighthouses Maria had posted on the bulletin board.*

"Can you see this one?" said Maria. "Can you see how tall it is? It is the tallest lighthouse in the world...this one, the small one had lots of windows."

The educator looked at the pictures and asked, "Could you tell me something else about the light houses? Could you write something about them?"

Maria went to the art shelf, took some large paper, and a bucket full of markers. She worked for a long period while her friends were getting ready to go outside. When she finished, she showed her teacher what she did: several tally marks with yellows circles on top.

"We saw lots of lighthouses," she said, "we saw seven."

Then, she added blue for the ocean and one bigger bright yellow circle.

"It was sunny," she said to the teacher.

Maria and her teacher posted the picture on the bulletin board for everybody to see. Maria smile with pride.

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Appendices

Appendix A

Letter of information
Early Learning Centre

July 2009

Dear Supervisor,

I am writing to ask the _____Early Learning Centre to consider participating in a research study. This study is entitled "Young Children Representing Numbers: Implications for Teaching".

The study will be conducted by myself, Gabriela Sanchez, and supervised with the knowledge and support of my supervisors, Dr. Martha Gabriel and Dr. Ray Doiron. I am an Early Childhood Educator and a Master of Education student. The study is a requirement for completition of a Master of Education degree at the University of Prince Edward Island.

As you are aware, little research has been done about early mathematics education. As a consequence, some mathematical contents in the early years have been oriented only to school preparation. This is the case, for example, of the expected use of digits by young children.

The proposed research is a qualitative study involving videotaping whole class and individual sessions of children from two to five years old. The videotaping and the interviews will explore and document children's processes of number representation. To assist me in the videotaping, I will have an assistant to operate the camera while I engaged with the children. The raw video will be ultimately edited to be used, as well as the study results, for future teaching purposes. While analyzing the video I may use some children's quotations to enrich the descriptions.

For the whole class sessions, all children of your Center in each age class will be invited to participate (the toddler class, the infant class, the preschool class and the kindergarten class). An information letter and consent form will need to be completed by the children's parents. Parent's consent for these children to participate is voluntary. If consent is given, I will conduct the sessions in the children's classrooms. Each whole class session will take approximately fifteen minutes.

A total of 8 children (1 boy and 1 girl in each of the four age groups) from your Center will be invited to participate in the individual sessions. These children must be identified by the Early Childhood Educators (ECEs) of your Centre as a representative of that age group. A separate information letter and consent form will need to be completed by the parents of the children chosen by your staff. Parents' consent for their child to participate is voluntary. If consent is given, I will conduct the individual sessions in a quiet room in your Center. Each individual session will take approximately fifteen minutes.

All of the sessions at your Centre will be completed in a one week period. On a daily basis, I will inform you orally about the session's progress.

Your Centre's participation is voluntary .You may choose to stop participating at any time, without any consequences. Even though videotaping is part of the study, I will ensure that either the Center's or the educators' or the children's names will be revealed.

If your Centre and staff agree to participate, I will be visiting each age group for an orientation time prior to the sessions. This orientation will take place at a convenient time for you and the ECEs. This will ensure that children are familiar with me before the data is collected.

I will be contacting you in a week to clarify any questions or concerns you or the ECEs might have. Times and a schedule for the sessions could be determined at that time.

Your Centre's participation in this study will contribute to build a better understanding of young children's own ways of representing numbers. I look forward to working with you and the ECEs and families at your Centre on this project. Once the project is completed, I will meet with you, your staff and parents to share the results of the study. Thanks for your collaboration.

Sincerely,

Gabriela Sanchez
M.Ed. student
(902) 626-3862
gsanchez@upei.ca

Consent form
Early Learning Centre's Supervisor

Study's title: Young Children Representing Numbers: Implications for Teaching.
Institution: University of Prince Edward Island
Faculty of Education

Conducted by:

Gabriela Sanchez
M.Ed.student

Supervised by

Ray Doiron, PhD
Director of the Centre for Education Research (CER)
Faculty of Education
University of Prince Edward Island.

Martha Gabriel, PhD
Faculty of Education
University of Prince Edward Island

I _____, understand that all children at _____Centre, ages two to five, have been invited to participate in a study entitled "Young Children Representing Numbers: Implications for Teaching". I understand that a group of 8 children have been invited to participate also in individual sessions of the same study. I understand that the sessions will involve demonstrating how young children represent, print and read numbers.

I understand that my name, the Centre's name, the ECEs' and the children's names will remain strictly confidential between the researcher, the researcher's supervisors, the educators and me.

II understand that parental consent is required for individual children's participation in the whole class sessions. I understand the whole class session will take place in the children's classroom and will take approximately fifteen minutes.

I understand that parental consent is required for children's participation in the individual sessions. I understand that individual sessions will take place in the room_____. I understand that each individual session will take approximately fifteen minutes.

I understand that all sessions will be videotaped. I understand that the edited video (as well as the results of the study) could be used in the future for teaching purposes in early childhood classes, conferences, workshops or publications. I understand that an assistant will operate the camera during all sessions. I understand that some children's quotations may be used for data analysis. I understand that my participation in this study will contribute to building a better understanding of young children's own ways of representing numbers.

I further understand that as a participant in this study, The Centre has several rights. I understand that the Centre's participation is strictly voluntary and that the Centre may discontinue its participation at any time. I understand that my name will be kept completely confidential and that under no circumstances will the Centre's name, the children's names, or my name be included in this report. I understand that the data collected by the researcher will be kept in her personal locked cabinet and password protected computer. I understand that the researcher and her supervisors are the only ones

who will have access to this information. I understand that the information will be kept confidential within the limits of the law. I understand that I can keep a copy of the signed and dated consent form.

Finally, I understand that the Centre, the ECEs and the parents will have access to a final report about the study.

I have read and understood the contents of this letter and agree to participate in the study.

Signed_____

Date_____

Researcher's signature:_____

Date:_____

Thank you for your willingness to participate in this study. Your participation is greatly appreciated. I may be contacted by phone at _____ or by email at _____ should you have any questions. Also, if you have any difficulties with, or wish to voice concern about any aspect of your participation in this study, or the ethical conduct of this study, you may contact the UPEI Research Ethics Board, for assistance at (902) 566-0637, Imacphee@upei.ca

Sincerely,
Gabriela Sanchez

Appendix B

Letter of information
Early Childhood Educator

July 2009
Dear Educator,

I am writing to ask you to considering participating in a research study entitled "Young Children Representing Numbers: Implications for Teaching".

The study will be conducted by myself, Gabriela Sanchez, and supervised with the knowledge and support of my supervisors, Dr. Martha Gabriel and Dr. Ray Doiron. I am an Early Childhood Educator and a Master of Education student. The study is an effort to complete the requirement of a Master of Education at the University of Prince Edward Island.

As you are aware, little research has been done about early mathematics education. As a consequence, some mathematical contents in the early years have been oriented only to school preparation. This is the case for example of the expected use of digits by young children.

The proposed research is a qualitative study involving videotaping whole class and individual sessions with children from two to five years of age. The videotaping and the interviews will explore and document children's processes of number representation. To assist me in the videotaping, I will have an assistant to operate the camera while I engaged with the children. The raw video will be ultimately be edited to be used, as well as the study results, for future teaching purposes. While analyzing the video I may use some children's quotations to enrich the descriptions.

For the whole class session all children in your class are invited to participate. An information letter and consent form will need to be completed by the children's parents. Parents' consent for these children to participate is voluntary. If consent is given, I will conduct the sessions in your classroom. I will be asking the children to count and to show me numbers in the classroom setting. This session will take approximately fifteen minutes. It is my intention to follow your class regular routine .The session will take place on the best time that suits your schedule and with your presence.

Two children from your class (a boy and a girl) will also be invited to participate in the individual sessions. I would like you to choose these children as

representative of a typical ____years old child. Please consider these children's cognitive, socio emotional and physical development. An information letter and consent form will need to be completed by the children's parents. Parents' consent for these children to participate is voluntary. If consent is given, I will conduct the sessions in the _____room at your Centre. During these sessions I will ask each child to represent and to print numbers. They will return to your classroom and after 20 minutes I will invite them again to come with me. During this opportunity I will observe the children's number's reading responses.

Your participation in this study is voluntary.You may decide to stop participating at any time, without any consequences. If your consent is given I will post the letters of information and consent forms for your children's parent in the children's lockers. A colour coded envelop will be posted in your classroom's door for parent to return their forms. I would really appreciate it if you could remind parents to do so.

If parents do not give consent for their child to participate in the whole class session, the child will continue with his/her regular class activities. If parents do not give consent for their child to participate in the individual session, I would like to ask you to choose another child whose parents could approve his /her participation.

Even though videotaping is part of the study, I will ensure that neither your name, the children's names nor the Centre's name will be revealed.

If you agree to participate I will be visiting each age group for an orientation time prior to the sessions. This orientation will take place at a convenient time for you and the ECEs. This will ensure that children are familiar with me before the data is collected.

I will be contacting you in a week to clarify questions or concerns that you might have. Times and a schedules for the sessions could be determined at that time.

Your participation in this study will contribute to building a better understanding of young children's own ways of representing numbers. I look forward to working with you and your children on this project. Once the project is completed, I will meet with all the participants to share the results of the study. Thank you for your collaboration.

Sincerely,

Gabriela Sanchez
M.Ed student
gsanchez@upei.ca

Consent form
Early Childhood Educator

Study's title: Young Children Representing Numbers: Implications for Teaching.
Institution: University of Prince Edward Island
Faculty of Education

Conducted by:

Gabriela Sanchez
M.Ed.student

Supervised by:

Ray Doiron, PhD
Director of the Centre for Education Research (CER)
Faculty of Education
University of Prince Edward Island.

Martha Gabriel, PhD
Faculty of Education
University of Prince Edward Island

I understand that the Centre _____has agreed to participate in the research study entitled "Young children representing numbers: Implications for Teaching".

I, _____, understand that all children from my class have been invited to participate in the whole class sessions of this study. I understand that two children of my class have been invited to participate also in individual sessions of the same study. I understand that my name, the Center's name and the children's names will remain strictly confidential between the researcher, the researcher's supervisors the Center's supervisor and me. I understand that the sessions will involve demonstrating how young children represent, print and read numbers.

I understand that the parents' consent is required for their child's participation in the whole class sessions. I understand the whole class session will take place in my classroom for approximately fifteen minutes. I understand that I will be present during this session. I understand that if parents do not give consent for their child to participate in the whole class session, I will make arrangements for the child to continue with his/her regular activities.

I understand that the parents' consent is required for their child's participation in the individual sessions. I understand that my expertise is required to choose two children from my class as representatives of a _____years old. I understand that individual sessions will take place in the _____ room. I understand that each individual session will take approximately fifteen minutes. I understand that if parents do not give consent for their child to participate in the individual session, I may choose another child from my class.

I understand that all sessions will be videotaped. I understand that the edited video (as well as the results of the study) could be used in the future for teaching purposes in early childhood classes, conferences, workshops or publications. I understand that an assistant will operate the camera during all sessions. I understand that some children's quotations may be used for data analysis. I understand that my participation in this study will contribute to building a better understanding of young children's own ways of representing numbers.

I further understand that as a participant in this study, I have several rights. I understand that my participation is strictly voluntary and that I may discontinue my participation at any time. I understand that my name will be kept completely confidential and that under no circumstances will the Centre's name, the children's names, or my name be include in this report. I understand that the data collected by the researcher will be kept in her personal locked cabinet and password protected computer. I understand that the researcher and her supervisors are the only ones who will have access to this information. I understand that I can keep a copy of the signed and dated consent form. I understand that the information will be kept confidential within the limits of the law. Finally, I understand that the Centre, the parents and I will have access to a final report about the study.

I have read and understood the contents of this letter and agree to participate in the study.

Signed_____

Date_____

Researcher's signature:_____

Date:_____

Thank you for your willingness to participate in this study. Your participation is greatly appreciated. I may be contacted by phone at _____ or by email at _____ should you have any questions. Also, if you have any difficulties with, or wish to voice concern about any aspect of your participation in this study, or the ethical conduct of this study, you may contact the UPEI Research Ethics Board, for assistance at (902) 566-0637, lmacphee@upei.ca

Sincerely,
Gabriela Sanchez

Appendix C

Letter of information
Whole class session- Parents/Guardians

July 2009

Dear Parent/Guardian:

The supervisor at the _____Centre has agreed to participate in a research study entitled "Young Children Representing Numbers: Implications for Teaching". I am writing to ask permission for your child to participate in this research study.

I am an Early Childhood Educator and a Master of Education student. I am this study to complete the requirements of the Master of Education degree at the University of Prince Edward Island. This study is undertaken with the knowledge and support of my supervisors, Dr. Martha Gabriel and Dr. Ray Doiron.

Your child is invited to participate in a whole class videotaped session. The session will take approximately fifteen minutes length. The session will take place in your child's classroom. I will be asking the children to count and to show me numbers in the classroom setting. Your child's educator will be present during the session.

Please be aware that even though videotaping is part of the study, your child's name won't be revealed on the data collected or in the final report. The raw video will be ultimately edited and used (as well as the study results) for future teaching purposes for example, conferences, seminars or workshops for early childhood educators. To assist me in the videotaping, I will have an assistant to operate the camera while I engaged with the children. For better descriptions of the video I may use some children's quotations.

Your consent for your child to participate is voluntary. You may decide for your child to stop participating at any time, without any consequences. If you do not agree for your child to participate in the study, the educator will make the necessary arrangement for your child to continue with classroom activities. I would appreciate it if you return the consent form by dropping it onto the colour coded envelope located on the door of your child's classroom.

Your child's participation in this study will contribute to building a better understanding of young children's own ways of representing numbers. Thank you for your collaboration.

Sincerely,

Gabriela Sanchez
M.Ed student

Consent form
Whole class session- Parent/Guardian

Study's title: Young Children Representing Numbers: Implications for Teaching.
Institution: University of Prince Edward Island
Faculty of Education

Conducted by:

Gabriela Sanchez
M.Ed.student

Supervised by

Ray Doiron, PhD
Director of the Centre for Education Research (CER)
Faculty of Education
University of Prince Edward Island.

Martha Gabriel, PhD
Faculty of Education
University of Prince Edward Island

I, _____, understand my child has been invited to participate in a study entitled "Young children representing numbers: Implications for Teaching". I understand that the Centre's _____ supervisor has agreed to participate in this study.

I understand that my child's name and my name will remain strictly confidential between the researcher, the researcher's supervisors, the center's supervisor and the educators.

I understand that the whole class session will take place in my child's classroom for approximately fifteen minutes. I understand that during this session my child will be asking to count and to show numbers to the researcher. I understand that the session will be videotaped and that the edited video as well as the results of the study could be used in the future for pedagogical purposes in early childhood classes, conferences, workshops or publications. I understand that an assistant will operate the camera in all sessions. I understand that my child's quotations could be used in the study. I understand that my child's participation in this study will contribute to building a better understanding of young children's own ways of representing numbers.

I further understand that as a parent, I have several rights. I understand that my child's participation is strictly voluntary and that he/she may discontinue his/her participation at any time without any consequences. I understand that my name or my child's will be kept completely confidential and that under no circumstances my child's name or my name will be include in the study report. I understand that the data collected by the researcher will be kept in her personal locked cabinet and password protected computer. I understand that the researcher and her supervisor are the only ones who could access to this information. I understand that the information will be kept confidential within the limits of the law.I understand that I can keep a copy of the signed and dated consent form.

Finally, I understand that the Centre and the parents will have access to a final report about the study.

I have read and understood the contents of this letter and agree to participate in the study.

Yes, I agree for my child _____to participate in the research study entitled "Yong Children Representing Numbers: Implications for Teaching."

Signed_____

Date_____

Researcher's signature:_____

Date:_____

No, I do not wish for my child _____to participate in the research study entitled "Yong Children Representing Numbers: Implications for Teaching."

Signed_____

Date_____

Researcher's signature:_____

Date:_____

Thank you for your willingness to participate in this study. Your participation is greatly appreciated. I may be contacted by phone at or by email at should you have any questions. Also, if you have any difficulties with, or wish to voice concern about any aspect of your participation in this study, or the ethical conduct of this study, you may contact the UPEI Research Ethics Board, for assistance at , Imacphee@upei.ca

Sincerely,
Gabriela Sanchez

Appendix D
Letter of information
Individual sessions- Parents/ Guardians

July 2009

Dear Parent:

The supervisor at the _____centre has agreed to participate in a research study entitled "Young Children Representing Numbers: Implications for Teaching". I am writing to ask permission for your child to participate in this research study.

I am an Early Childhood Educator and a Master of Education student. This research is an effort to complete the requirement of the Master of Education at the University of Prince Edward Island and is undertaken with the knowledge and support of my supervisors, Dr. Martha Gabriel and Dr. Ray Doiron.

Your child is invited to participate in an individual videotaped session. This session will take approximately fifteen minutes. The session will take place in the ____room of the Centre. During this session I will be asking your child to represent, to print and to read numbers.

Please be aware that even though videotaping is part of the study, your child's name won't be revealed on the data collected or in the final report. The raw video will be ultimately edited to be used, (as well as the study results) for future teaching purposes in for example, conferences, seminars or workshops for early childhood educators. To assist me in the videotaping, I will have an assistant to operate the camera while I engaged with the children. For better descriptions of the video I may use some children's quotations.

Your consent for your child to participate is voluntary and you may decide that you want your child to stop participating at any time, without any consequences. I would appreciate it if you return the consent form by placing it in the colour coded envelope located on the door of your child's classroom.

Your child's participation in this study will contribute to building a better understanding of young children's own ways of representing numbers. Thank you for your collaboration.

Sincerely,
Gabriela Sanchez
M.Ed student-

Consent form
Individual sessions- Parent/ Guardian

Study's title: Young Children Representing Numbers: Implications for Teaching.
Institution: University of Prince Edward Island
Faculty of Education

Conducted by:

Gabriela Sanchez
M.Ed.student

Supervised by
Ray Doiron, PhD
Director of the Centre for Education Research (CER)
Faculty of Education
University of Prince Edward Island.

Martha Gabriel, PhD
Faculty of Education
University of Prince Edward Island

Parent's /Guardian's individual Consent form

I, _____, understand my child has been invited to participate in a study entitled "Young Children Representing Numbers: Implications for Teaching". I understand that the Centre's _____ supervisor has agreed to participate in this study.

I understand that my child's name and my name will remain strictly confidential between the researcher, the researcher's supervisors, the center's supervisor and the educators.

I understand that the individual session will take place in the _____room. I understand that my child will be interviewed for approximately fifteen minutes. I understand that my child will be asked to represent, to print and to read numbers.

I understand that the sessions will be videotaped and that the edited video as well as the results of the study could be used in the future for teaching purposes in early childhood classes, conferences, workshops or publications. I understand that an assistant will operate the camera during all sessions. I understand that my child's quotations may be used in the study. I understand that my child's participation in this study will contribute to building a better understanding of young children's own ways of representing numbers.

I further understand that as a parent, I have several rights .I understand that my child's participation is strictly voluntary and that he/she may discontinue his/her participation at any time. I understand that my name or my child's will be kept completely confidential and that under no circumstances my child's name or my name will be include in the study report. I understand that the data collected by the researcher will be kept in her personal locked cabinet and password protected computer. I understand that the researcher and her supervisor are the only ones who could access to this information. I understand that I can keep a copy of the signed and dated consent form. I understand that the information will be kept confidential within the limits of the law.

Finally, I understand that the Centre and the parents will have access to a final report about the study.

have read and understood the contents of this letter and agree to participate in the study..
Yes, I agree for my child _____to participate in the individual session of the research study entitled "Yong Children Representing Numbers: Implications for Teaching."
Signed_____
Date_____
Researcher's signature:_____
Date:_____

No, I do not wish for my child _____to participate in the individual sessions of the research study entitled "Yong Children Representing Numbers: Implications for Teaching."
Signed_____
Date_____
Researcher's signature:_____
Date:_____
Signed_____

Thank you for your willingness to participate in this study. Your participation is greatly appreciated. I may be contacted by phone at or by email at should you have any questions. Also, if you have any difficulties with, or wish to voice concern about any aspect of your participation in this study, or the ethical conduct of this study, you may contact the UPEI Research Ethics Board, for assistance at

Sincerely,
Gabriela Sanchez